Decision Procedures – Homework 2

Discussed on Tuesday, 28th July, 2016.

Exercise 2.1 Proving equivalence of programs

Show that the two programs do the same, using uninterpreted functions

```
int factorial3_a (int in) {
    int i, out_a=in;
    for (i=0; i<2; i++)
        out_a=out_a*(in-i);
    return out_a;
}
```

```
int factorial3_b (int in) {
    int out_b;
    out_b= in * (in-1) * (in-2)
    return out_b
}
```

(a) (b)

Solution:

Outcome of the compiling Process

- in1 = in - 1
- in2 = in - 2
- a = in * in1
- out_a = out1_a * in2
- out_b = in * in1 * in2

Verification Condition

- in1 = in - 1 ∧
- in2 = in - 2 ∧
- a = in * in1 ∧
- out_a = out1_a * in2
- ⇒ out_b = in * (in - 1) * (in - 2)
- ⇒ out_b = in * (in - 1) * (in - 2)

Abstracted Version

- out_a = G(G(in, F(in, 1)), F(in, 2))
- out_b = G(G(in, F(in, 1)), F(in, 2))

Exercise 2.2 Ackermann’s Reduction

Apply Ackermann’s Reduction to the following term:

\[ x_1 = x_2 \land y_1 = y_2 \land F(x_1, F(x_1, G(F(x_1, y_1)))) = F(x_2, F(x_2, G(F(x_2, y_2)))) \]

Solution:

- Indexing
  - \[ f_1 := F(x_1, y_1) \]
  - \[ g_1 := G(f_1) \]
  - \[ f_2 := F(x_1, g_1) \]
  - \[ f_3 := F(x_1, f_2) \]
  - \[ f_4 := F(x_2, y_2) \]
  - \[ g_2 := G(f_4) \]
  - \[ f_5 := F(x_2, g_2) \]
  - \[ f_6 := F(x_2, f_5) \]

- Transform flat
  - \[ flat^E := x_1 = x_2 \Rightarrow f_3 = f_6 \]
Exercise 2.3  Congruence Closure

Apply the congruence closure algorithm to the following terms:

(a) $\varphi^U_a := x_1 = x_2 \land x_2 \neq x_3 \land x_3 = x_4 \land x_4 = x_5 \land F(x_1) \neq F(x_3)$

(b) $\varphi^U_b := x_1 = x_2 \land x_2 = x_3 \land x_3 \neq x_4 \land F(x_1, x_2) = F(x_2, x_4)$

(c) $\varphi^U_c := x_1 = x_2 \land x_2 \neq x_3 \land F(x_3) = F(x_2) \land \neg (F(x_1) \neq G(x_3) \lor G(x_2) = G(x_3))$

(d) $\varphi^U_d := F(F(F(x))) = x \land F(F(F(x)))) = x \land F(x) \neq x$

Solution:

(a) $\{x_1, x_2\} \{x_3, x_4, x_5\} \{F(x_1)\} \{F(x_3)\}$
   \Rightarrow \text{Satisfiable}

(b) $\{x_1, x_2, x_3\} \{x_4\} \{F(x_1, x_2), F(x_2, x_4)\}$
   \Rightarrow \text{Unsatisfiable}

(c) $\{x_1, x_2\} \{x_3\} \{F(x_1), F(x_2), G(x_3)\} \{G(x_3)\}$
   \Rightarrow \text{Satisfiable}

(d) $\{x, F(x), F(F(F(x))), F(F(F(F(x))))\}$
   \Rightarrow \text{Unsatisfiable}