Einführung in die funktionale Programmierung
Valentin Hauner – WS 2013/14

Reducing lists with foldr, foldl
foldr

• from the Prelude:

\[
\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b
\]

\[
\text{foldr } f z \; [] = z
\]

\[
\text{foldr } f z \; (x:xs) = f \; x \; (\text{foldr } f z \; xs)
\]
foldr

• from the Prelude:
  
  foldr :: (a -> b -> b) -> b -> [a] -> b
  foldr f z [] = z
  foldr f z (x:xs) = f x (foldr f z xs)

• example:
  
  foldr (+) 0 [1,2,3] =
foldr

• from the Prelude:

\[ \text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b \]
\[ \text{foldr} \ f \ z \ [\] = z \]
\[ \text{foldr} \ f \ z \ (x:xs) = f \ x \ (\text{foldr} \ f \ z \ xs) \]

• example:

\[ \text{foldr} \ (+) \ 0 \ [1,2,3] = \]
\[ 1 + (\text{foldr} \ (+) \ 0 \ [2,3]) = \]
foldr

- from the Prelude:

  \[ \text{foldr} :: (a -> b -> b) -> b -> [a] -> b \]
  \[ \text{foldr} \ f \ z \ [] = z \]
  \[ \text{foldr} \ f \ z \ (x:xs) = f \ x \ (\text{foldr} \ f \ z \ xs) \]

- example:

  \[ \text{foldr} \ (+) \ 0 \ [1,2,3] = \]
  \[ 1 + (\text{foldr} \ (+) \ 0 \ [2,3]) = \]
  \[ 1 + (2 + (\text{foldr} \ (+) \ 0 \ [3])) = \]
foldr

- from the Prelude:
  
  \[
  \text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b \\
  \text{foldr} \; f \; z \; [] = z \\
  \text{foldr} \; f \; z \; (x:xs) = f \; x \; (\text{foldr} \; f \; z \; xs)
  \]

- example:
  
  \[
  \text{foldr} \; (+) \; 0 \; [1,2,3] = \\
  1 + (\text{foldr} \; (+) \; 0 \; [2,3]) = \\
  1 + (2 + (\text{foldr} \; (+) \; 0 \; [3])) = \\
  1 + (2 + (3 + (\text{foldr} \; (+) \; 0 \; []))) =
  \]
foldr

- from the Prelude:

\[
foldr :: (a \to b \to b) \to b \to [a] \to b
\]

\[
foldr \ f \ z \ [] = z
\]

\[
foldr \ f \ z \ (x:xs) = f \ x \ (foldr \ f \ z \ xs)
\]

- example:

\[
foldr \ (+) \ 0 \ [1,2,3] =
\]

\[
1 + (foldr \ (+) \ 0 \ [2,3]) =
\]

\[
1 + (2 + (foldr \ (+) \ 0 \ [3])) =
\]

\[
1 + (2 + (3 + (foldr \ (+) \ 0 \ []))) =
\]

\[
1 + (2 + (3 + 0)) =
\]

6
foldl
foldl

• from the Prelude:

\[
\text{foldl} :: (a \to b \to a) \to a \to [b] \to a
\]

\[
\text{foldl} \ f \ z \ [] \ = \ z
\]

\[
\text{foldl} \ f \ z \ (x:xs) \ = \ \text{foldl} \ f \ (f \ z \ x) \ xs
\]
foldl

• from the Prelude:

  \[
  \texttt{foldl} :: (a \to b \to a) \to a \to [b] \to a \\
  \texttt{foldl}\ f\ z\ [] = z \\
  \texttt{foldl}\ f\ z\ (x:xs) = \texttt{foldl}\ f\ (f\ z\ x)\ xs
  \]

• example:

  \[
  \texttt{foldl}\ (+)\ 0\ [1,2,3] =
  \]

  \[
  \begin{array}{c}
  \texttt{foldl f z} \\
  \vdots \hspace{2cm} \vdots \\
  5 \hspace{2cm} 1 \\
  4 \hspace{2cm} f \\
  3 \hspace{2cm} f \\
  2 \hspace{2cm} f \\
  1 \hspace{2cm} f \\
  \end{array}
  \]
foldl

• from the Prelude:

foldl :: (a -> b -> a) -> a -> [b] -> a
foldl f z []     = z
foldl f z (x:xs) = foldl f (f z x) xs

• example:

foldl (+) 0 [1,2,3] =
foldl (+) (0 + 1) [2,3] =
foldl

• from the Prelude:

\[
\text{foldl} :: (a \to b \to a) \to a \to [b] \to a \\
\text{foldl } f \ z \ \text{[]} \ = \ z \\
\text{foldl } f \ z \ (x:x:s) = \text{foldl } f \ (f \ z \ x) \ x:s
\]

• example:

\[
\text{foldl } (+) \ 0 \ [1,2,3] = \\
\text{foldl } (+) \ (0 + 1) \ [2,3] = \\
\text{foldl } (+) \ ((0 + 1) + 2) \ [3] =
\]
foldl

• from the Prelude:

\[
\text{foldl} :: (a \to b \to a) \to a \to [b] \to a
\]
\[
\text{foldl } f \ z \ [] = z
\]
\[
\text{foldl } f \ z \ (x:xs) = \text{foldl } f \ (f \ z \ x) \ xs
\]

• example:

\[
\text{foldl } (+) \ 0 \ [1,2,3] =
\]
\[
\text{foldl } (+) \ (0 + 1) \ [2,3] =
\]
\[
\text{foldl } (+) \ (((0 + 1) + 2) \ [3] =
\]
\[
\text{foldl } (+) \ (((0 + 1) + 2) + 3) \ [] =
\]
foldl

• from the Prelude:

$$\text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a$$

$$\text{foldl} \ f \ z \ [] = z$$

$$\text{foldl} \ f \ z \ (x:x:s) = \text{foldl} \ f \ (f \ z \ x) \ x:s$$

• example:

$$\text{foldl} \ (+) \ 0 \ [1,2,3] =$$

$$\text{foldl} \ (+) \ (0 + 1) \ [2,3] =$$

$$\text{foldl} \ (+) \ ((0 + 1) + 2) \ [3] =$$

$$\text{foldl} \ (+) \ (((0 + 1) + 2) + 3) \ [] =$$

$$(((0 + 1) + 2) + 3 =$$

6
foldr/foldl

foldl (+) 0 [1,2,3] =
1 + (foldl (+) 0 [2,3]) =
1 + (2 + (foldl (+) 0 [3])) =
1 + (2 + (3 + (foldr (+) 0 []))) =
1 + (2 + (3 + 0)) =
6

foldr (+) 0 [1,2,3] =
1 + (foldr (+) 0 [2,3]) =
1 + (2 + (foldr (+) 0 [3])) =
1 + (2 + (3 + (foldl (+) 0 []))) =
1 + (2 + (3 + 0)) =
6

=> foldr and foldl produce the same result if the operation used is associative
foldr

- from the Prelude:
  \[
  \text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b
  \]
  \[
  \text{foldr } f z \text{ [] } = z
  \]
  \[
  \text{foldr } f z \text{ (x:xs) } = f \text{ x (foldr } f z \text{ xs)}
  \]

- example:
  \[
  \text{foldr } (-) \text{ 0 [1,2,3] } =
  \]
foldr

• from the Prelude:

\[ \text{foldr} :: (a \to b \to b) \to b \to [a] \to b \]
\[ \text{foldr} \ f \ z \ [] = z \]
\[ \text{foldr} \ f \ z \ (x:xs) = f \ x \ (\text{foldr} \ f \ z \ xs) \]

• example:

\[ \text{foldr} \ (-) \ 0 \ [1,2,3] = \]
\[ 1 - (\text{foldr} \ (-) \ 0 \ [2,3]) = \]
foldr

- from the Prelude:

  \[
  \text{foldr} :: (a \to b \to b) \to b \to [a] \to b \\
  \text{foldr } f \ z \ [\] \ = \ z \\
  \text{foldr } f \ z \ (x:xs) \ = \ f \ x \ (\text{foldr } f \ z \ xs)
  \]

- example:

  \[
  \text{foldr } (-) \ 0 \ [1,2,3] \ = \ \\
  1 \ - \ (\text{foldr } (-) \ 0 \ [2,3]) \ = \ \\
  1 \ - \ (2 \ - \ (\text{foldr } (-) \ 0 \ [3])) =
  \]
foldr

- from the Prelude:

  \[
  \text{foldr} :: (a \to b \to b) \to b \to [a] \to b \\
  \text{foldr} \ f \ z \ [] = z \\
  \text{foldr} \ f \ z \ (x:xs) = f \ x \ (\text{foldr} \ f \ z \ xs)
  \]

- example:

  \[
  \text{foldr} \ (-) \ 0 \ [1,2,3] = \\
  1 - (\text{foldr} \ (-) \ 0 \ [2,3]) = \\
  1 - (2 - (\text{foldr} \ (-) \ 0 \ [3])) = \\
  1 - (2 - (3 - (\text{foldr} \ (-) \ 0 \ []))) = 
  \]
foldr

• from the Prelude:

\[
\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b
\]

\[
\text{foldr } f \ z \ [] = z
\]

\[
\text{foldr } f \ z \ (x:xs) = f \ x \ (\text{foldr } f \ z \ xs)
\]

• example:

\[
\text{foldr } (-) \ 0 \ [1,2,3] =
1 \ - \ (\text{foldr } (-) \ 0 \ [2,3]) =
1 \ - \ (2 \ - \ (\text{foldr } (-) \ 0 \ [3])) =
1 \ - \ (2 \ - \ (3 \ - \ (\text{foldr } (-) \ 0 \ []))) =
1 \ - \ (2 \ - \ (3 \ - \ 0)) =
2
\]
foldl

- from the Prelude:
  \[
  \text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
  \]

  \[
  \text{foldl } f \ z \ [] = z
  \]

  \[
  \text{foldl } f \ z \ (x:xs) = \text{foldl } f \ (f \ z \ x) \ xs
  \]

- example:

  \[
  \text{foldl } (-) \ 0 \ [1,2,3] =
  \]

\[
\text{foldl } f z
\]

\[
\begin{array}{c}
1
/\   \\
2
/   \\
3
/   \\
4
/   \\
5
/   \\
[]
/   \\
z
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
4
/   \\
f
/   \\
f
/   \\
f
/   \\
5
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
2
/   \\
f
/   \\
3
/   \\
f
/   \\
4
/   \\
f
/   \\
5
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
1
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\
f
/   \\

foldl

- from the Prelude:
  \[
  \text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
  \]
  \[
  \text{foldl} f z [] = z
  \]
  \[
  \text{foldl} f z (x:xs) = \text{foldl} f (f z x) xs
  \]

- example:
  \[
  \text{foldl} (-) 0 [1,2,3] =
  \]
  \[
  \text{foldl} (-) (0 - 1) [2,3] =
  \]
foldl

from the Prelude:

\[
\text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a \\
\text{foldl} \ f \ z \ [] = z \\
\text{foldl} \ f \ z \ (x : xs) = \text{foldl} \ f \ (f \ z \ x) \ xs
\]

example:

\[
\text{foldl} \ (-) \ 0 \ [1, 2, 3] = \\
\text{foldl} \ (-) \ (0 - 1) \ [2, 3] = \\
\text{foldl} \ (-) \ ((0 - 1) - 2) \ [3] =
\]
foldl

- from the Prelude:

  \[\text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a\]

  \[\text{foldl} \, f \, z \, [] = z\]

  \[\text{foldl} \, f \, z \, (x:xs) = \text{foldl} \, f \, (f \, z \, x) \, xs\]

- example:

  \[\text{foldl} \, (\cdot) \, 0 \, [1,2,3] =\]

  \[\text{foldl} \, (\cdot) \, (0 - 1) \, [2,3] =\]

  \[\text{foldl} \, (\cdot) \, ((0 - 1) - 2) \, [3] =\]

  \[\text{foldl} \, (\cdot) \, (((0 - 1) - 2) - 3) \, [] =\]
foldl

• from the Prelude:

\[
foldl :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
\]

\[
foldl \ f \ z \ [] = z
\]

\[
foldl \ f \ z \ (x:xs) = foldl \ f \ (f z x) \ xs
\]

• example:

\[
foldl \ (-) \ 0 \ [1,2,3] =
\]

\[
foldl \ (-) \ (0 - 1) \ [2,3] =
\]

\[
foldl \ (-) \ ((0 - 1) - 2) \ [3] =
\]

\[
foldl \ (-) \ (((0 - 1) - 2) - 3) \ [] =
\]

\[
(((0 - 1) - 2) - 3 =
\]

-6
### foldr/foldl

<table>
<thead>
<tr>
<th>foldr ((-)) 0 [1,2,3]</th>
<th>foldl ((-)) 0 [1,2,3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 - (foldr ((-)) 0 [2,3]))</td>
<td>(foldl ((-)) (0 (-) 1) [2,3] =)</td>
</tr>
<tr>
<td>(1 - (2 - (foldr ((-)) 0 [3]))))</td>
<td>(foldl ((-)) ((0 (-) 1) (-) 2) [3] =)</td>
</tr>
<tr>
<td>(1 - (2 - (3 - (foldr ((-)) 0 []))))</td>
<td>(foldl ((-)) (((0 (-) 1) (-) 2) (-) 3) [] =)</td>
</tr>
<tr>
<td>(1 - (2 - (3 - 0)))</td>
<td>(((0 (-) 1) (-) 2) (-) 3 =)</td>
</tr>
<tr>
<td>2</td>
<td>(-6)</td>
</tr>
</tbody>
</table>

=> foldr and foldl do **not** produce the same result if the operation used is **not** associative
foldr: cons

• from the Prelude:

\[
\text{foldr} :: (a \to b \to b) \to b \to [a] \to b
\]
\[
foldr \ f \ z \ [] = z
\]
\[
foldr \ f \ z \ (x:xs) = f \ x \ (foldr \ f \ z \ xs)
\]

• example:

\[
foldr \ (:) \ [] \ [1,2,3] =
\]
foldr: cons

• from the Prelude:

\[
\text{foldr} :: (a \to b \to b) \to b \to [a] \to b
\]

\[
\begin{align*}
\text{foldr } f \; z \; [] & = z \\
\text{foldr } f \; z \; (x:xs) & = f \; x \; (\text{foldr } f \; z \; xs)
\end{align*}
\]

• example:

\[
\text{foldr } (:) \; [] \; [1,2,3] =
\]

\[
1 : (\text{foldr } (:) \; [] \; [2,3]) =
\]
foldr: cons

• from the Prelude:

\[
\text{foldr} :: (a \to b \to b) \to b \to [a] \to b
\]

\[
foldr \ f \ z \ [] = z
\]

\[
foldr \ f \ z \ (x:xs) = f \ x \ (foldr \ f \ z \ xs)
\]

• example:

\[
foldr \ (:) \ [] \ [1,2,3] =
\]

\[
1 : (foldr \ (:) \ [] \ [2,3]) =
\]

\[
1 : (2 : (foldr \ (:) \ [] \ [3])) =
\]
foldr: cons

• from the Prelude:

\[
\text{foldr} :: (a \rightarrow b \rightarrow b) \rightarrow b \rightarrow [a] \rightarrow b
\]

\[
\text{foldr} \ f \ z \ [\] = z
\]

\[
\text{foldr} \ f \ z \ (x:xs) = f \ x \ (\text{foldr} \ f \ z \ xs)
\]

• example:

\[
\text{foldr} \ (:) \ [] \ [1,2,3] =
\]

\[
1 : (\text{foldr} \ (:) \ [] \ [2,3]) =
\]

\[
1 : (2 : (\text{foldr} \ (:) \ [] \ [3])) =
\]

\[
1 : (2 : (3 : (\text{foldr} \ (:) \ [] \ []))) =
\]
foldr: cons

- from the Prelude:

\[
\text{foldr} :: (a \to b \to b) \to b \to [a] \to b
\]
\[
\text{foldr } f \ z \ [\!] = z
\]
\[
\text{foldr } f \ z \ (x:x:s) = f \ x \ (\text{foldr } f \ z \ x:s)
\]

- example:

\[
\text{foldr } (:) \ [\!] \ [1,2,3] =
\]
1 : (foldr (:) [] [2,3]) =
1 : (2 : (foldr (:) [] [3]))) =
1 : (2 : (3 : (foldr (:) [] []))) =
1 : (2 : (3 : [])) =

[1,2,3]
foldl: cons

• from the Prelude:

```haskell
foldl :: (a -> b -> a) -> a -> [b] -> a
foldl f z []      = z
foldl f z (x:xs)  = foldl f (f z x) xs
```

• example:

```haskell
foldl (:) [] [1,2,3]
```

*type error! why?
foldl: cons

• from the Prelude:

\[
\text{foldl} :: (a \to b \to a) \to a \to [b] \to a
\]

\[
\text{foldl } f \; z \; [] = z
\]

\[
\text{foldl } f \; z \; (x:xs) = \text{foldl } f \; (f \; z \; x) \; xs
\]

• example:

\[
\text{foldl } (:) \; [] \; [1,2,3] = \text{foldl } (\lambda a \; b \to a:b) \; [] \; [1,2,3]
\]

*type error! why?*
foldl: cons

• from the Prelude:

\[
\begin{align*}
\text{foldl} & :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a \\
\text{foldl } f \ z \ [\] & = z \\
\text{foldl } f \ z \ (x:xs) & = \text{foldl } f \ (f \ z \ x) \ xs
\end{align*}
\]

• example:

\[
\begin{align*}
\text{foldl } (:) \ [\] \ [1,2,3] & = \text{foldl } (\lambda \ a \ b \rightarrow a:b) \ [\] \ [1,2,3] \\
\text{type error! why?}
\end{align*}
\]

\[
\begin{align*}
\text{foldl } (\lambda \ xs \ x \rightarrow x:xs) \ [\] \ [1,2,3] & =
\end{align*}
\]
foldl: cons

- from the Prelude:
  
  ```haskell
  foldl :: (a -> b -> a) -> a -> [b] -> a
  foldl f z []     = z
  foldl f z (x:xs) = foldl f (f z x) xs
  ```

- example:

  ```haskell
  foldl (:) [] [1,2,3] = foldl (\a b -> a:b) [] [1,2,3]
  type error! why?

  foldl (\xs x -> x:xs) [] [1,2,3] =
  foldl (\xs x -> x:xs) (1:[]) [2,3] =
  ```
foldl: cons

• from the Prelude:

\[
foldl \, \colon \, (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
\]

\[
foldl \, f \, z \, [] \quad = \quad z
\]

\[
foldl \, f \, z \, (x:xs) \quad = \quad foldl \, f \, (f \, z \, x) \, xs
\]

• example:

\[
foldl \, (:) \, [] \, [1,2,3] \quad = \quad foldl \, (\lambda \, a \, b \rightarrow a:b) \, [] \, [1,2,3]
\]

\textbf{type error! why?}

\[
foldl \, (\lambda \, xs \, x \rightarrow x:xs) \, [] \, [1,2,3] \quad =
\]

\[
foldl \, (\lambda \, xs \, x \rightarrow x:xs) \, (1:[]) \, [2,3] \quad =
\]

\[
foldl \, (\lambda \, xs \, x \rightarrow x:xs) \, (2:(1:[])) \, [3] \quad =
\]
foldl: cons

- from the Prelude:
  
  \[
  \text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
  \]

  \[
  \text{foldl} \ f \ z \ [] \quad = \quad z
  \]

  \[
  \text{foldl} \ f \ z \ (x:xs) \quad = \quad \text{foldl} \ f \ (f \ z \ x) \ xs
  \]

- example:

  \[
  \text{foldl} \ (:) \ [] \ [1,2,3] \quad = \quad \text{foldl} \ (\a \ b \rightarrow \a:\b) \ [] \ [1,2,3]
  \]

  \text{type error! why?}

  \[
  \text{foldl} \ (\xs \ x \rightarrow \ x:xs) \ [] \ [1,2,3] \quad = \\
  \text{foldl} \ (\xs \ x \rightarrow \ x:xs) \ (1:[]) \ [2,3] \quad = \\
  \text{foldl} \ (\xs \ x \rightarrow \ x:xs) \ (2:(1:[])) \ [3] \quad = \\
  \text{foldl} \ (\xs \ x \rightarrow \ x:xs) \ (3:(2:(1:[]))) \ [] \quad = 
  \]
foldl: cons

• from the Prelude:

\[
\text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a
\]
\[
foldl f z [] = z
\]
\[
foldl f z (x:xs) = foldl f (f z x) xs
\]

• example:

\[
\text{foldl} (:) [] [1,2,3] = \text{foldl} (\lambda a b \rightarrow a:b) [] [1,2,3]
\]

*type error! why?*

\[
foldl (\lambda xs x \rightarrow x:xs) [] [1,2,3] =
\]
\[
foldl (\lambda xs x \rightarrow x:xs) (1:[]) [2,3] =
\]
\[
foldl (\lambda xs x \rightarrow x:xs) (2:(1:[])) [3] =
\]
\[
foldl (\lambda xs x \rightarrow x:xs) (3:(2:(1:[]))) [] =
\]
\[
(3:(2:(1:[]))) = [3,2,1]
\]
foldl: cons

• from the Prelude:

\[
\text{foldl} :: (a \rightarrow b \rightarrow a) \rightarrow a \rightarrow [b] \rightarrow a \\
\text{foldl} f z [] = z \\
\text{foldl} f z (x:xs) = \text{foldl} f (f z x) xs
\]

• example:

\[
\text{foldl } (:) [] [1,2,3] = \text{foldl } (\lambda a b \rightarrow a:b) [] [1,2,3] \\
\text{type error! why?}
\]

\[
\text{foldl } (\lambda xs x \rightarrow x:xs) [] [1,2,3] = \\
\text{foldl } (\lambda xs x \rightarrow x:xs) (1:[]) [2,3] = \\
\text{foldl } (\lambda xs x \rightarrow x:xs) (2:(1:[])) [3] = \\
\text{foldl } (\lambda xs x \rightarrow x:xs) (3:(2:(1:[]))) [] = \\
(3:(2:(1:[])))) = [3,2,1]
\]

=> tail recursion, will be efficiently compiled as a loop
foldl: cons

- from the Prelude:
  \[
  \text{foldl} :: (a \to b \to a) \to a \to [b] \to a \\
  \text{foldl} \ f \ z \ [] = z \\
  \text{foldl} \ f \ z \ (x:xs) = \text{foldl} \ f \ (f \ z \ x) \ xs
  \]

- example:
  \[
  \text{foldl} \ (:) \ [] \ [1,2,3] = \text{foldl} \ (\lambda a \ b \to a:b) \ [] \ [1,2,3] \\
  \text{type error! why?}
  \]

  \[
  \text{foldl} \ (\lambda xs \ x \to x:xs) \ [] \ [1,2,3] = \\
  \text{foldl} \ (\lambda xs \ x \to x:xs) \ (1:[]) \ [2,3] = \\
  \text{foldl} \ (\lambda xs \ x \to x:xs) \ (2:(1:[])) \ [3] = \\
  \text{foldl} \ (\lambda xs \ x \to x:xs) \ (3:(2:(1:[]))) \ [] = \\
  (3:(2:(1:[])))) = \\
  [3,2,1]
  \]

  => tail recursion, will be efficiently compiled as a loop
  => does not work with infinite lists! why?
Infinite lists

• example:

```haskell
> let xs = [1..]
> xs
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26
27,28,29,30,31,32,33,34,35,36,37,38,39,40,41, . . .]
```
Infinite lists

• example:

```hs
> let xs = [1..]
> xs
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26
  27,28,29,30,31,32,33,34,35,36,37,38,39,40,41, . . . ]

head ( foldr (:) [] xs ) =
```
Infinite lists

• example:

```haskell
> let xs = [1..]
> xs
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26
  27,28,29,30,31,32,33,34,35,36,37,38,39,40,41, .. .]

head ( foldr (:) [] xs ) =
head ( 1 : (foldr (:) [] [2..]) ) =
Infinite lists

• example:

```haskell
> let xs = [1..]
> xs
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26
  27,28,29,30,31,32,33,34,35,36,37,38,39,40,41, . . . ]
```

```haskell
head ( foldr (:) [] xs ) =
head ( 1 : (foldr (:) [] [2..]) ) =
1
```
Infinite lists

• example:

```haskell
> let xs = [1..]
> xs
[1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26
  27,28,29,30,31,32,33,34,35,36,37,38,39,40,41, . . . ]

head ( foldr (:) [] xs ) =
head ( 1 : (foldr (:) [] [2..]) ) =
1

head ( foldl f [] xs ) => counts to infinity!
```
References


(all web resources requested on November 11th, 2013)