**Goals**  First steps in Simpl and concepts and rules of semantics and Hoare logic.

For the non-Isabelle exercises on this sheet, please answer each question in a complete sentence.

**Exercise 1 [3] Basic notions**

1. What is the idea of a *shallow embedding*?
2. Name some part of Simpl which is embedded shallowly and some part which is embedded deeply.
3. The *Soundness* of the Hoare logic for Simpl is expressed by the following theorem. Explain this proposition in a sentence.

\[ \Gamma, \Theta \vdash P \c Q, A \equiv \Gamma, \Theta \mid P \c Q, A \]

**Exercise 2 [4] Semantics**

1. Explain how the following rules model a conditional expression:

\[
\begin{align*}
[s \in b; \Gamma \vdash \langle c_1, \text{Normal } s \rangle \Rightarrow t] & \Rightarrow \Gamma \vdash \langle \text{Cond } b \ c_1 \ c_2, \text{Normal } s \rangle \Rightarrow t \\
[s \notin b; \Gamma \vdash \langle c_2, \text{Normal } s \rangle \Rightarrow t] & \Rightarrow \Gamma \vdash \langle \text{Cond } b \ c_1 \ c_2, \text{Normal } s \rangle \Rightarrow t
\end{align*}
\]

2. The *Basic* command encodes state updates. Explain the rule:

\[ \Gamma \vdash \langle \text{Basic } f, \text{Normal } s \rangle \Rightarrow \text{Normal } (fs) \]

3. How do the rules above rely on a shallow embedding-approach?

4. The Simpl state type is:

```
datatype xstate = Normal state | Abrupt state | Fault | Stuck
```

How can the state *Abrupt* ever be reached? What is the meaning of this state?

**Exercise 3 [3] Hoare Logic**

1. The Hoare logic provides the following rule for state updates. Why does it match the semantic given above?

\[ \Gamma, \Theta \vdash \{s. f \ s \in Q\} (\text{Basic } f) Q, A \]

2. For the branches of a conditional statement, one wants to have the test result as additional knowledge. How is this realized in each of the following rules?

\[
\begin{align*}
[\Gamma, \Theta \vdash (P \cap b) c_1 Q, A; \Gamma, \Theta \vdash (P \cap -b) c_2 Q, A] & \Rightarrow \Gamma, \Theta \vdash P (\text{Cond } b \ c_1 \ c_2) Q, A \\
\Gamma, \Theta \vdash (P \cap b) c P, A; & \Rightarrow \Gamma, \Theta \vdash P (\text{While } b \ c) (P \cap -b), A
\end{align*}
\]

3. The classic rule for While loops is stated below. Why is the usual name *invariant* for *I* a sensible choice?

\[ [P \subseteq I; \Gamma, \Theta \vdash (I \cap b) c I, A; \ I \cap -b \subseteq Q] \Rightarrow \Gamma, \Theta \vdash P (\text{While } b \ c) Q \]

**Exercise 4 [3] Getting Started with Simpl**

For this and later exercises you need the Simpl package from the *Archive of Formal Proofs*. See the *Sheet04.thy* for the Simpl exercises.