Exercise 1 (Confluence & Commutation)

Show: If $\rightarrow_1$ and $\rightarrow_2$ are confluent, and if $\rightarrow_1^*$ and $\rightarrow_2^*$ commute, then $\rightarrow_{12} := \rightarrow_1 \cup \rightarrow_2$ is also confluent.

Exercise 2 (Local Commutation)

Show: If $t_2 \leftarrow s \rightarrow t_1 \Rightarrow \exists u. t_2 \rightarrow^* u \leftarrow t_1^* \leftarrow t_1$, then $\rightarrow_1^*$ and $\rightarrow_2^*$ commute.

Here $\rightarrow^*$ denotes the reflexive closure of $\rightarrow$, i.e.:

$\rightarrow^* := \rightarrow \cup \rightarrow^0$

Exercise 3 (Strong Confluence)

A relation $\rightarrow$ is said to be strongly confluent iff:

$t_2 \leftarrow s \rightarrow t_1 \Rightarrow \exists u. t_2 \rightarrow^* u \leftarrow t_1$

Show that every strongly confluent relation is also confluent.

Homework 4 (Semi-Confluence)

A relation $\rightarrow$ is said to be semi-confluent iff:

$t_2 \leftarrow s \rightarrow t_1 \Rightarrow \exists u. t_2 \rightarrow^* u \leftarrow t_1$

Show that $\rightarrow$ is semi-confluent if and only if it is confluent.

Homework 5 (Diamond Property & Normal Forms)

Show that if $\rightarrow$ has the diamond property, every element is either in normal form or has no normal form.

Homework 6 (Weak Diamond Property)

Assume that $\rightarrow$ has the following weaker diamond property:

$t_2 \leftarrow s \rightarrow t_1 \land t_1 \neq t_2 \Rightarrow \exists u. t_2 \rightarrow u \leftarrow t_1$.

a) Is it still the case that every element is either in normal form or has no normal form?

b) Show that if $t$ has a normal form, then all its reductions to its normal form have the same length.