Homework is due on June 29th, before the tutorial.

Exercise 1 (H) (Completion)

Complete

{f(g(f(x))) \approx x}

to a convergent term rewriting system!

Exercise 2 (H) (Linear Term Rewriting Systems)

A rewrite rule \( l \rightarrow r \) is called left-linear if every variable in \( l \) occurs exactly once. Similarly, \( l \rightarrow r \) is called right-linear if every variable in \( r \) occurs exactly once. A rule is linear if it is both right- and left-linear. We say that a term rewriting system is linear if it contains only linear rules.

Show:

a) Every linear term rewriting system \( R \) that has no critical pairs is confluent. (You must give a self-contained proof; do not simply apply Corollary 6.3.11 from the textbook! \( Hint: \) Show that \( R \) is strongly confluent.)

b) If \( R \) is a linear term rewriting system, and for every critical pair \((t_1, t_2)\) there exists \( t_0 \) such that \( t_1 \overset{\rightarrow}{\rightarrow} t_0 \overset{\leftarrow}{\leftarrow} t_2 \), then \( R \) is confluent. \( Hint: \) Extend your proof of a).

Exercise 3 (T) (Confluence)

Let \( R \) be the following term rewriting system:

{\( f(x, x) \rightarrow a, c \rightarrow g(c), g(x) \rightarrow f(x, g(x)) \)}

Is \( R \) confluent? Justify your answer.

Exercise 4 (T) (Completion)

Complete

{\( f(g(f(x))) \approx f(g(x)) \)}

to a convergent term rewriting system!

Exercise 5 (T) (Completion)

You are given the following equations:

\[ E = \{ 1 \cdot x \approx x, \ x \cdot 1 \approx x, \ i(x) \cdot (x \cdot y) \approx y \} \]

Construct a term rewriting system that decides equality with respect to \( E \).