Exercise 29 (Most General Unifier)

Let $S$ and $T$ be unification problems. Moreover, let $\sigma$ be a most general unifier for $S$ and $\theta$ be a most general unifier for $\sigma(T)$. Show that $\theta\sigma$ is a most general unifier for $S \cup T$.

Exercise 30 (Equivalence Relations)

Prove that the notions of a equivalence relation and a partition coincide.

Reminder: A set of sets $P$ is called a partition of a set $M$ iff all elements of $P$ are pairwise disjoint, $P$ does not contain the empty set, and $\bigcup P = M$.

Exercise 31 (Termination)

A term rewriting system $R$ is called right reduced, if for all $(l \rightarrow r) \in R$, the right hand side $r$ is irreducible. Show that every right reduced and right ground term rewriting system terminates.

Hint: Consider the positions in the term at which rules from $R$ may be applied, and specify a suitable order on terms. Is there a simpler way to get this lemma as a corollary from a lemma that was presented in the lecture?

Homework 32 (Compactness)

Prove that every satisfiable set of equations over a finite set of variables contains a finite subset that has the same solutions.

Note that equations are interpreted in the term algebra.

Hint: Select a countable subset of the set of equations.

Homework 33 (Deciding Termination for Right-Ground TRSs)

In the lecture, we discussed a procedure to decide termination of right-ground term rewriting systems. It is important that we use a breadth-first search strategy, as you shall demonstrate in this exercise.

a) Given the following procedure that uses a depth-first approach:

Input A right ground term rewriting system $R = \{l_1 \rightarrow r_1, \ldots, l_n \rightarrow r_n\}$.
**Procedure** Enumerate all reduction sequences that start with \( r_1 \), in depth-first order.

If one of those sequences contains \( r_1 \) as a subterm, output *non-terminating.*

Otherwise continue with the sequences starting at \( r_2 \), and so on. If all right hand sides have been processed, output *terminating.*

Find a right-ground term rewriting system such that the above procedure does not terminate.

b) Determine whether the following rewriting systems terminate using the breadth-first approach:

\[
R_1 = \{ f(x, x) \rightarrow f(a, b), b \rightarrow c \} \\
R_2 = \{ f(x, x) \rightarrow f(a, b), b \rightarrow a, b \rightarrow c \}
\]

c) Implement the correct algorithm in Haskell. More instructions can be found on the lecture website.