Exercise 1.1.  [Short Questions]
Let $M$ be a set of formulas, and let $F$ and $G$ be formulas. Which of the following assertions hold?

1. If $F$ satisfiable then $M \models F$
2. $F$ is valid iff $\top \models F$
3. If $\models F$ then $M \models F$
4. If $M \models F$ then $M \cup \{G\} \models F$
5. $M \models F$ and $M \models \neg F$ cannot hold simultaneously
6. If $M \models G \rightarrow F$ and $M \models G$ then $M \models F$

Exercise 1.2.  [Coincidence Lemma]
Assume that for all atomic formulas $A_i$ in $F$, $\mathcal{A}(A_i) = \mathcal{A'}(A_i)$. Show that

$\mathcal{A} \models F$ iff $\mathcal{A'} \models F$

Exercise 1.3.  [Semantic Proof]
Let $\models F \rightarrow G$ where $F$ and $G$ do not share any atoms. Show that then either $F$ is unsatisfiable or $G$ is an tautology (or both). Hint: you may want to use the previous result.

Exercise 1.4.  [Satisfiability Algorithms]
Check the following formulas for satisfiability using one of the algorithms seen in the lecture:

- $F_1 = (\neg A \lor \neg D \lor B) \land D \land \neg B \land E \land (\neg D \lor \neg E \lor C)$
- $F_2 = (A \rightarrow C) \land (C \rightarrow E) \land (\top \rightarrow B) \land (C \land A \rightarrow D) \land (B \rightarrow A) \land (B \rightarrow E) \land (D \land E \rightarrow \bot)$
- $F_3 = (A \rightarrow E) \land (B \rightarrow \bot) \land (C \rightarrow B) \land (\top \rightarrow A) \land (E \land B \rightarrow C) \land (C \rightarrow D)$
Announcement: You have to achieve a minimum combined score of 50% on your homeworks to be granted a 0.3/0.4 grade bonus on the final exam, provided that the exam is passed.

Homework 1.1. [CNF and DNF] (5 points)
Use the rewriting-based procedure from the lecture to convert the following formulas $F$ and $G$ first to NNF, and then to CNF and DNF. Document each rewriting step.

$$F = \neg \neg \neg (A_1 \land \neg (A_2 \lor A_3))$$
$$G = \neg (A_1 \lor \neg A_2 \land (A_3 \lor A_1))$$

Homework 1.2. [Consequence] (5 points)
Let $M$ be a finite set of formulas, and $F_1, \ldots, F_n$ be formulas. Consider the two statements:

1. $\{F_1, \ldots, F_n\} \subseteq M$
2. $M \models \bigwedge_{i=1}^n F_i$

Prove that 1) implies 2). Are the statements even equivalent? Proof or counterexample!

Homework 1.3. [Duality and NNF] (5 points)
Let $F$ be a formula in NNF. Let $F'$ be the formula that is constructed from $F$ by replacing every $\land$ by $\lor$, every $\lor$ by $\land$, every $\neg A_i$ by $A_i$, and every $A_i$ by $\neg A_i$. Show that, for every assignment $A$, we have: If $A \models F$, then $A \not\models F'$!

Homework 1.4. [Small Parity Formulas] (5 points)
We define the size $|F|$ of a formula $F$ by counting the number of operator symbols and atomic formulas. Formally:

$$|A_i| = 1 \quad |\neg F| = 1 + |F| \quad |F \land G| = 1 + |F| + |G| \quad |F \lor G| = 1 + |F| + |G|$$

Your task is to construct a family of formulas $\{F_i \mid i \in \mathbb{N}\}$ such that $F_n$ contains the atomic formulas $A_1, \ldots, A_n$, and we have for every assignment $A$:

$$A \models F_n \text{ if and only if } |\{A_i \mid 1 \leq i \leq N \text{ and } A(A_i) = 1\}| \text{ is odd}$$

Moreover, the size of your formula must be at most quadratic in $n$, i.e., $|F_n| = O(n^2)$. For simplicity, you may assume that $n$ is a power of two. Justify your construction (both, correctness and size)!