Exercise 2.1.  [Predicate Logic]
   a) Specify a satisfiable formula $F$, such that for all models $\mathcal{A}$ of $F$, we have $|U_{\mathcal{A}}| \geq 3$.
   b) Can you also specify a satisfiable formula $F$, such that for all models $\mathcal{A}$ of $F$, we have $|U_{\mathcal{A}}| \leq 3$?

Exercise 2.2.  [Resolution Completeness]
   a) Does $F \models C$ imply $F \vdash_{\text{Res}} C$? Proof or counterexample!
   b) Can you prove $F \models C$ by resolution?

Exercise 2.3.  [Resolution of Horn-Clauses]
   Can the resolvent of two Horn-clauses be a non-Horn clause?

Exercise 2.4.  [Optimizing Resolution]
   We call a clause $C$ trivially true if $A_i \in C$ and $\neg A_i \in C$ for some atom $A_i$. Show that the resolution algorithm remains complete if it does not consider trivially true clauses for resolution.

Exercise 2.5.  [Finite Axiomatization]
   Let $M_0$ and $M$ be sets of formulas. $M_0$ is called axiom schema for $M$, iff for all assignments $\mathcal{A}$: $\mathcal{A} \models M_0$ iff $\mathcal{A} \models M$.

   A set $M$ is called finitely axiomatized iff there is a finite axiom scheme for $M$.

   a) Are all sets of formulas finitely axiomatized? Proof or counterexample? b) Let $M = (F_i)_{i \in \mathbb{N}}$ be a set of formulas, such that for all $i$: $F_{i+1} \models F_i$, and not $F_{i+1} \models F_i$. Is $M$ finitely axiomatized?
Homework 2.1. [Definitonal CNF] (3 points)
Calculate the definitonal CNF of the following formula:

$$(A_1 \lor (A_2 \land \neg A_3)) \lor A_4$$

Homework 2.2. [Definitonal DNF] (5 points)
We call formulas $F$ and $F'$ equivalent if

$$\models F \iff \models F'$$

First show that

$F[G/A]$ and $(A \leftrightarrow G) \rightarrow F$ are equivalent

for any formulas $F$ and $G$ and any atom $F$, provided that $A$ does not occur in $G$. Now argue that for every formula $F$ of size $n$ there is an equivalent DNF formula $G$ of size $O(n)$.

Homework 2.3. [Compactness Theorem] (5 points)
Suppose every subset of $S$ is satisfiable. Show that then

every subset of $S \cup \{F\}$ is satisfiable or
for any formula $F$.

Homework 2.4. [Compactness and Validity] (2 points)
We say that a set of formulas $S$ is valid if every $F$ in $S$ is valid. Prove or disprove:

$$S \text{ is valid iff every finite subset of } S \text{ is valid}$$

Homework 2.5. [Resolution] (5 points)
Use the resolution procedure to decide if the following formulas are satisfiable. Show your work (by giving the corresponding DAG or linear derivation)!

1. $$(\neg A_1 \land A_2) \land (\neg A_1 \lor A_3) \land (A_1 \lor \neg A_2 \lor A_3)$$
2. $$A_2 \land (\neg A_3 \lor A_1) \land (\neg A_1 \lor A_2) \land (\neg A_1) \land (\neg A_2 \lor A_3)$$