Exercise 1.1.  [Short Questions]
Let $M$ be a set of formulas, and let $F$ and $G$ be formulas. Which of the following assertions hold?

1. If $F$ satisfiable then $M \models F$
2. $F$ is valid iff $\top \models F$
3. If $\models F$ then $M \models F$
4. If $M \models F$ then $M \cup \{G\} \models F$
5. $M \models F$ and $M \models \neg F$ cannot hold simultaneously
6. If $M \models G \rightarrow F$ and $M \models G$ then $M \models F$

Exercise 1.2.  [Coincidence Lemma]
Assume that for all atomic formulas $A_i$ in $F$, $A_i(A_i) = A_i'(A_i)$. Show that

$$A \models F \iff A' \models F$$

Exercise 1.3.  [Semantic Proof]
Let $\models F \rightarrow G$ where $F$ and $G$ do not share any atoms. Show that then $F$ is unsatisfiable or $G$ is a tautology (or both).  Hint: you may want to use the previous result.

Exercise 1.4.  [Satisfiability Algorithms]
Check the following formulas for satisfiability using one of the algorithms seen in the lecture:

- $F_1 = (\neg A \lor \neg D \lor B) \land D \land \neg B \land E \land (\neg D \lor \neg E \lor C)$
- $F_2 = (A \rightarrow C) \land (C \rightarrow E) \land (\top \rightarrow B) \land (C \land A \rightarrow D) \land (B \rightarrow A) \land (B \rightarrow E) \land (D \land E \rightarrow \bot)$
- $F_3 = (A \rightarrow E) \land (B \rightarrow \bot) \land (C \rightarrow B) \land (\top \rightarrow A) \land (E \land B \rightarrow C') \land (C \rightarrow D)$
Homework 1.1.  [CNF and DNF] (6 points)
Use the rewriting-based procedure from the lecture to convert the following formulas \( F \) and \( G \) first to NNF, and then to CNF and DNF. Document each rewriting step.

\[
F = \neg(\neg A_1 \land \neg\neg(A_2 \lor A_3)) \quad G = (A_1 \lor A_2 \lor A_3) \land (\neg A_1 \lor \neg A_2)
\]

Homework 1.2.  [Basic equivalences] (8 points)
Let \( F \) and \( G \) be formulas. Are the following statements equivalent? Proof or counterexample!

1. \( \models F \leftrightarrow G \)
2. \( F \equiv G \)

How about these two statements?

1. \( F \) is valid
2. \( F \equiv \top \)

Homework 1.3.  [Efficient CNF satisfiability check] (6 points)
In general, solving satisfiability for CNF formula is a hard problem. Consider the special case where clauses may only contain up to two literals. Give an efficient algorithm to check satisfiability.