Exercise 6.1.  [Unused Bound Variables]
For this exercise, we first define an alternative way to evaluate a formula in a structure, based on arithmetic and set operations.

\[
\begin{align*}
\mathcal{A}(\neg F) &= 1 - \mathcal{A}(F) \\
\mathcal{A}(F \lor G) &= \max(\mathcal{A}(F), \mathcal{A}(G)) \\
\mathcal{A}(F \land G) &= \min(\mathcal{A}(F), \mathcal{A}(G)) \\
\mathcal{A}(\exists x \ F) &= \max\{\mathcal{A}[d/x](F) \mid d \in U_A\} \\
\mathcal{A}(\forall x \ F) &= \min\{\mathcal{A}[d/x](F) \mid d \in U_A\}
\end{align*}
\]

The evaluation for predicates and terms remain unchanged.

Equipped with this definition, prove the equivalence \( \exists x F \equiv F \) where \( x \) does not occur in \( F \).

**Hint:** Adapt the coincidence lemma for propositional logic (exercise 1.1) to predicate logic.

Exercise 6.2.  [Substitution Lemma]
Consider the following statement: “If \( F \equiv F' \), then \( F[t/x] \equiv F'[t/x] \).” Proof or counterexample.

Exercise 6.3.  [Skolem Form]
Convert the following formula into – in order – a rectified formula, RPF and Skolem form.

\[ P(x) \land \forall x \ (Q(x) \land \forall x \exists y \ P(f(x, y))) \]

Exercise 6.4.  [Herbrand Models]
Given the formula

\[ F = \forall x \forall y(P(f(x), g(y)) \land \neg P(g(x), f(y))) \]

1. Specify a Herbrand model for \( F \).
2. Specify a Herbrand structure suitable for \( F \), which is not a model of \( F \).
Homework 6.1.  [Skolem Form]  (6 points)
Convert the following formulas into – in order – a rectified formula, RPF and Skolem form.

1. \( \forall x \exists y \forall z \exists w (\neg P(a, w) \lor Q(f(x), y)) \)
2. \( \forall z \exists y (P(x, g(y), z) \lor \neg \forall x Q(x)) \)

Homework 6.2.  [Invalid Herbrand Models]  (8 points)
Recall the fundamental theorem from the lecture: “Let \( F \) be a closed formula in Skolem form. Then \( F \) is satisfiable iff it has a Herbrand model”.

Explain “what goes wrong” if the precondition is violated: when \( F \) is not closed or not in Skolem form. Describe both cases.

Homework 6.3.  [Proof of the Fundamental Theorem]  (6 points)
Recall the proof of the fundamental theorem from the lecture. Give the omitted proof for the base case (slide 6, \( A(G) = T(G) \)).