Exercise 8.1.  [Decidability]

1. Resolution for first-order logic is sound and complete.
2. Satisfiability and validity for first-order logic are undecidable.

How do you reconcile these two facts?

Exercise 8.2.  [Ground Resolution]
Use ground resolution to prove that the following formula is valid:

\[(\forall x P(x, f(x))) \rightarrow \exists y P(c, y)\]

Exercise 8.3.  [Barber Paradox]
Consider the following facts:

1. Every barber shaves those who do not shave themselves.
2. No barber shaves anyone who shaves themselves.

Show with resolution that there are no barbers by resolution.
**Homework 8.1.**  **[Restricted Resolution]**  (8 points)
In the resolution procedure as defined in the lecture slides, we can unify arbitrarily many literals from two clauses. Consider a modified resolution procedure, where exactly one literal is picked. We add another rule ("collapsing rule"): For a clause $C = \{L_1, \ldots, L_n\}$, where \{L_i, L_j\} can be unified using a mgu $\delta$, add another clause $C' = (C - L_i)\delta$.

For example, given the clause

$$C = \{\neg W(x), \neg W(f(y)), T(x, y), \neg W(f(c))\}$$

we can apply the collapsing rule as follows:

$$L_1 = \neg W(x), L_2 = \neg W(f(y)), \delta = \{x \mapsto f(y)\}, C' = \{\neg W(f(y)), T(f(y), y), \neg W(f(c))\}$$

(Note that there are multiple possible ways to apply the collapsing rule to $C$.)

Prove that our modified resolution calculus, including collapsing rule, can be simulated by the original resolution calculus, and vice versa.

**Homework 8.2.**  **[Resolution]**  (8 points)
Show with resolution that:

1. $\forall x(\neg R(x) \rightarrow R(f(x))) \rightarrow \exists x(R(x) \land R(f(x)))$ is valid
2. $\exists x(P(x) \land \neg P(f(f(x)))) \land \forall x(P(x) \rightarrow P(f(x)))$ is unsatisfiable

**Homework 8.3.**  **[Equality]**  (4 points)
We consider how to model equality in predicate logic. In the lecture slides, the following axiom schema for congruence is used:

$$\begin{align*}
Eq(x_i, y) \\
\frac{Eq(f(x_1, \ldots, x_i, \ldots, x_n), f(x_1, \ldots, y, \ldots, x_n))}{Eq(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n))}
\end{align*}$$

Assume that this schema is replaced by:

$$\begin{align*}
Eq(x_1, y_1) & \quad \cdots \quad Eq(x_n, y_n) \\
\frac{Eq(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n))}{Eq(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n))}
\end{align*}$$

Reflexivity, symmetry and transitivity stay unchanged. Show that the above modified schemas is equivalent to the schemas from the lecture.

*Hint:* Simulate the modified schema with the original one and vice versa.