Exercise 11.1 Using the VCG

Use the VCG to prove correct a multiplication and a square root program:

 definition MUL :: com
 lemma "—
  {As. 0 ≤ s "y" ∧ s=sorig}
  MUL
  {As. s "x" = s "y" ∧ (∀ v. v∉{"z","c"} → s v = sorig v)}"

definition "SQRT ≡
  "r" ::= N 0;;
  "s" ::= N 1;;
  WHILE (Not (Less (V "x") (V "s"))) DO 
    "r" ::= Plus (V "s") (N 1);;
    "s" ::= Plus (V "s") (V "r");;
    "s" ::= Plus (V "s") (V "r");;
    "r" ::= Plus (V "s") (N 1)
  )"

 lemma "—
  {As. s=sorig ∧ s "x" ≥ 0}
  SQRT
  {As. (s "r")² ≤ s "x" ∧ s "x" < (s "r"+1)² ∧ (∀ v. v∉{"s","r"} → s v = sorig v)}"

Exercise 11.2 Total Correctness

Prove total correctness of the multiplication and square root program

Rotated rule for sequential composition:

lemmas Seq.bwd = Hoare_Total.Seq[rotated]

Prove the following syntax-directed conditional rule (for total correctness):

lemma IfT:
  assumes "— t {P1} c₁ {Q}" and "— t {P2} c₂ {Q}"
shows \( \vdash_t \{ \lambda s. (\text{beal } b \ s \rightarrow P_1 s) \land (\neg \text{beal } b \ s \rightarrow P_2 s) \} \text{ IF } b \text{ THEN } c_1 \text{ ELSE } c_2 \{ Q \} \)

lemmas hoareT_rule[intro?] = Seq_bwd Hoare_Total.Assign Hoare_Total.Assign' IfT

Homework 11  Be Original!

Submission until Tuesday, 12 January 2016, 10:00am. (20 regular points, plus bonus points for nice submissions)

Think up a nice formalization yourself, for example

- Prove some interesting result about graph/automata/formal language theory
- Formalize some results from mathematics
- Prove some results from program optimization
- ...

In case you don’t have a good idea, here are some further inspirations: Register machines, register allocation, non-trivial IMP-programs, IMP + \{ procedures, arrays, etc \}

You should set yourself a time limit before starting your project. Also incomplete/unfinished formalizations are welcome and will be graded!

Please comment your formalization well, such that we can see what it does/is intended to do.

You are welcome to discuss your plans with the tutor before starting your project.

Merry Christmas!