Aufgabe 9.1. [Stack-Append Lemma] (10 points)

1. Prove the stack-append lemma from the lecture. For a pushdown-system $M = (P, \Gamma, \text{Act}, p_0, \gamma_0, \Delta)$, $l \in \text{Act}^*$, $p, p' \in P$ and $w, w', v, v' \in \Gamma^*$ we have

$$pw \xrightarrow{l \ast} p'w' \implies pwv \xrightarrow{l \ast} p'w'v$$

Hint: Induction on the length of $l$.

2. Give a counterexample to the reverse proposition, i.e., specify a pushdown system and configurations such that

$$pwv \xrightarrow{l \ast} p'w'v \implies pw \xrightarrow{l \ast} p'w'$$

does not hold.

Aufgabe 9.2. [Labels] (10 points)
Recall: For a PDS $M$, we defined a tree automaton $A_M$, that describes the set of execution trees of $M$. For any execution $p_0\gamma_0 \xrightarrow{l \ast} p'w'$ of $M$, we have a corresponding execution tree $t \in L(A_M)$ such that $c(t) = p'w'$.

Here, the function $c : \text{XN} \rightarrow \text{PT}^*$ extracts the reached configuration from an execution tree. We now want to also extract the sequence of actions, i.e., we want to define a function $a : \text{XN} \rightarrow \text{Act}^*$, such that the following theorem holds:

$$p_0\gamma_0 \xrightarrow{l \ast} p'w' \iff \exists t \in L(A_M) \land c(t) = p'w' \land a(t) = l$$

Specify such a function $a$. You do not need to prove the theorem. Hint: You may need to specify an auxiliary function for returning execution trees.