Semantics of Programming Languages
Exercise Sheet 11

Exercise 11.1 Sign Analysis

Instantiate the abstract interpretation framework to a sign analysis over the lattice pos, zero, neg, any, where pos abstracts positive values, zero abstracts zero, neg abstracts negative values, and any abstracts any value.

datatype sign = Pos | Zero | Neg | Any

Exercise 11.2 AI for Conditionals

Our current constant analysis does not regard conditionals. For example, it cannot figure out, that after executing the program x:=2; IF x<2 THEN x:=2 ELSE x:=1, x will be constant.

In this exercise, we extend our abstract interpreter with a simple analysis of boolean expressions. To this end, modify locale Val.semilattice in theory Abs_Int0.thy as follows:

- Introduce an abstract domain 'bv for boolean values, add, analogously to num' and plus' also functions for the boolean operations and for less.
- Modify Abs_Int0 to accommodate for your changes.
Homework 11.1  Complete Lattices

*Submission until Monday, January 27, 10:00am.*

*Note:* This homework could be a typical exam exercises. It is not done in Isabelle, so either submit your answer in a text block in the theory file that you turn in or submit it on paper before the start of the next tutorial.

Which of the following ordered sets are complete lattices?

- \( \mathbb{N} \), the set of natural numbers \( \{0, 1, 2, 3, \ldots \} \) with the usual order.
- \( \mathbb{N} \cup \{\infty\} \), the set of natural numbers plus infinity, with the usual order and \( n < \infty \) for all \( n \in \mathbb{N} \).
- A finite set \( A \) with a total order \( \leq \) on it.
- \( \mathbb{B}^* \), the set of all lists of booleans \( \{[], [\text{True}], [\text{False}], \ldots \} \), with prefix order: \( a \leq b \iff \exists c. b = a \# c \).
- The set of all lists of a certain length \( n \) if a point-wise ordering is used.

For each ordered set, prove (no Isabelle proof) that it is a complete lattice or give a counter example. Note: You do not need to prove the order properties, only the properties that constitute complete lattice.

Homework 11.2  Using the VCG

*Submission until Monday, January 27, 10:00am.*

Instantiate the abstract interpretation framework to find out which variables have values in the range \( \{-128 \ldots 127\} \), i.e. fit into one byte.

Start your development from \( \text{HOL−IMP.Abs_Int1_parity} \). You do not need to show termination.

Test your framework on the test cases for constant propagation from \( \text{HOL−IMP.Abs_Int Tests} \).