Derivatives of WS1S Formulas

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Logic-Automaton Connection

WS1S

$$\varphi = \mathrm{T} \mid \mathrm{F} \mid x \in X \mid x < y \mid \varphi \vee \varphi \mid \neg\varphi \mid \exists x.\, \varphi \mid \exists X.\, \varphi$$

finite

Logic-Automaton Connection

WS1S

$\forall \mathrm{I}.\ \ \mathrm{I} \models \varphi \iff \mathrm{I} \models \psi?$

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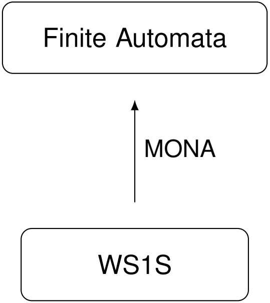
Logic-Automaton Connection



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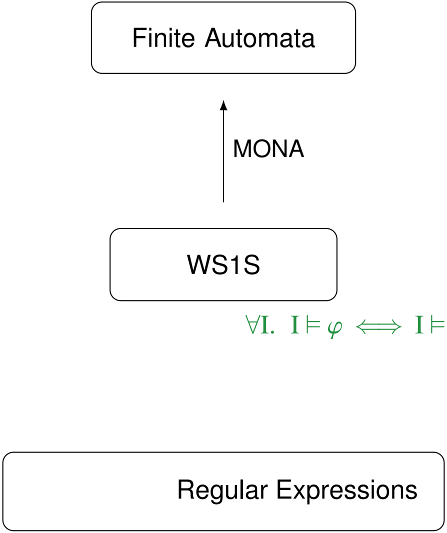
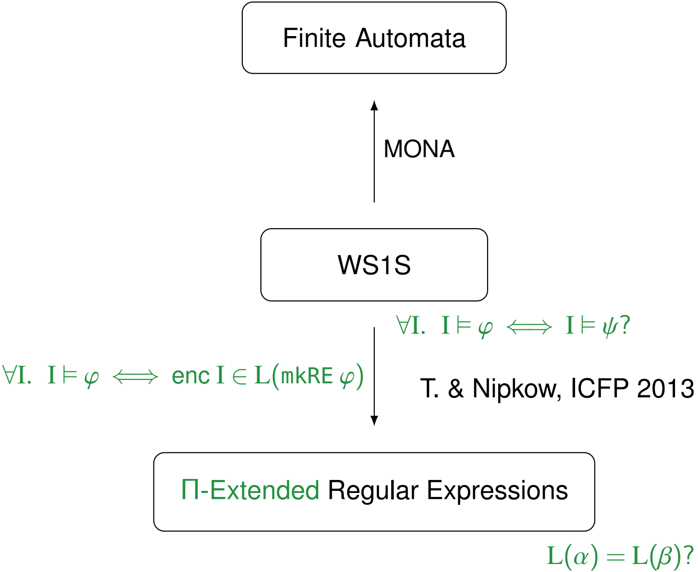


$\forall \mathrm{I}.\ \ \mathrm{I} \models \varphi \iff \mathrm{I} \models \psi?$

$\mathrm{L}(\alpha) = \mathrm{L}(\beta)?$

Logic-Automaton Connection



$a^* \stackrel{?}{=} \varepsilon + a \cdot a^*$ for $\Sigma = \{a, b\}$

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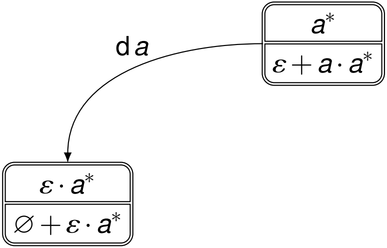
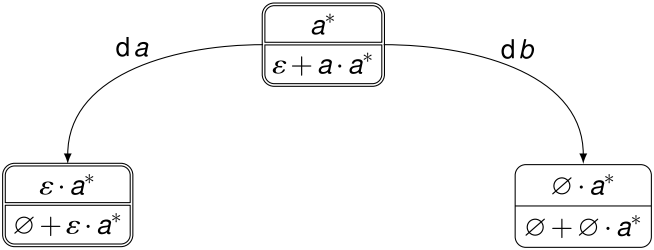


Brzozowski derivative

$$\mathrm{d} : \text{letter} \to \text{regex} \to \text{regex}$$

$$\mathrm{L}(\mathrm{d}\, a\, r) = \{w \mid aw \in \mathrm{L}(r)\}$$

$$a^* \stackrel{?}{\equiv} \varepsilon + a \cdot a^* \text{ for } \Sigma = \{a, b\}$$



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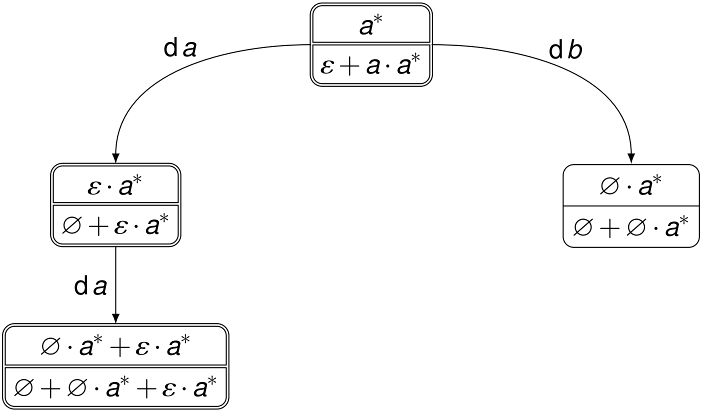

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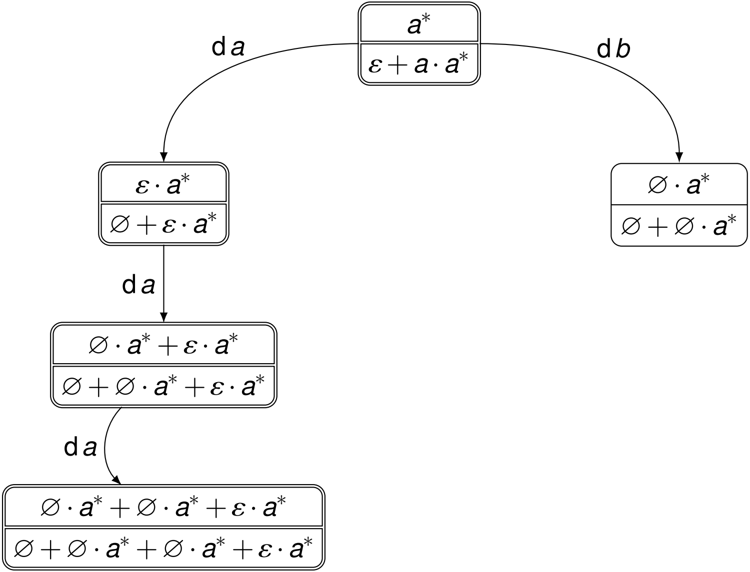

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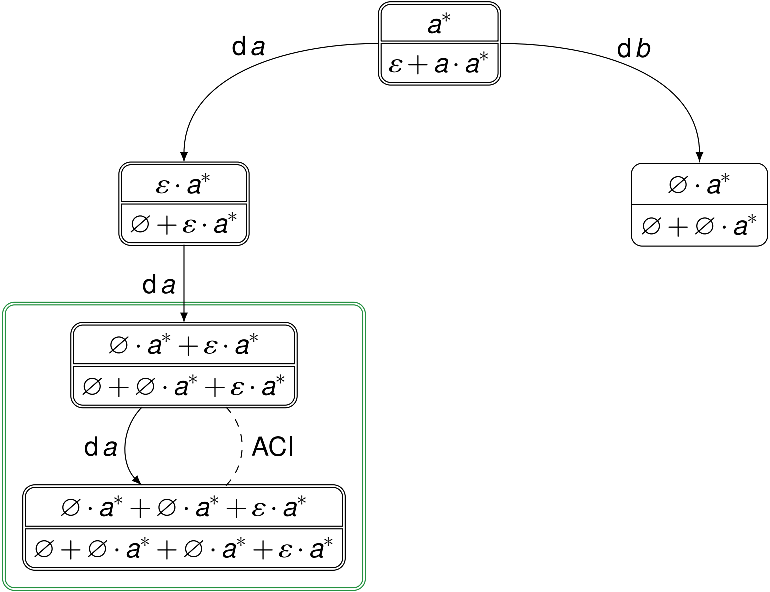

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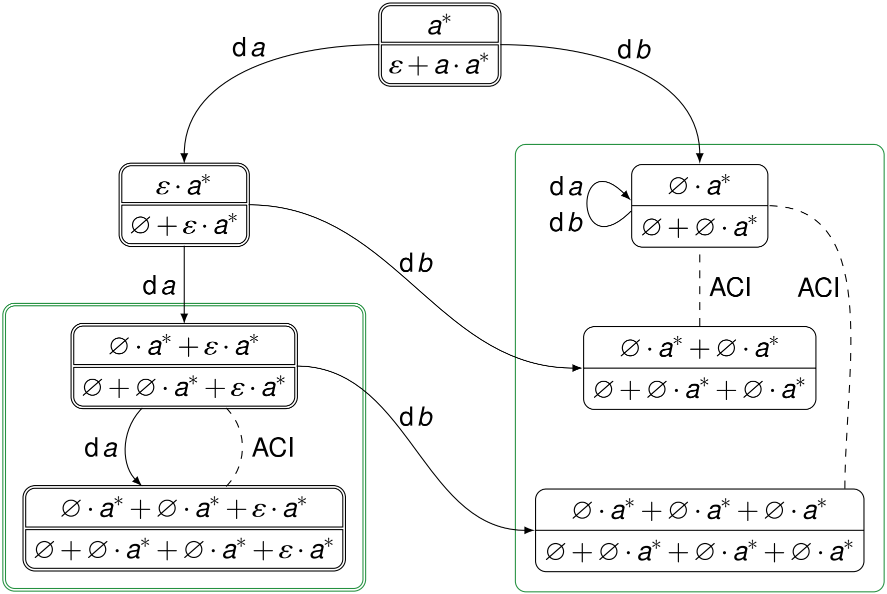

Key ingredients: Derivative + Acceptance Test

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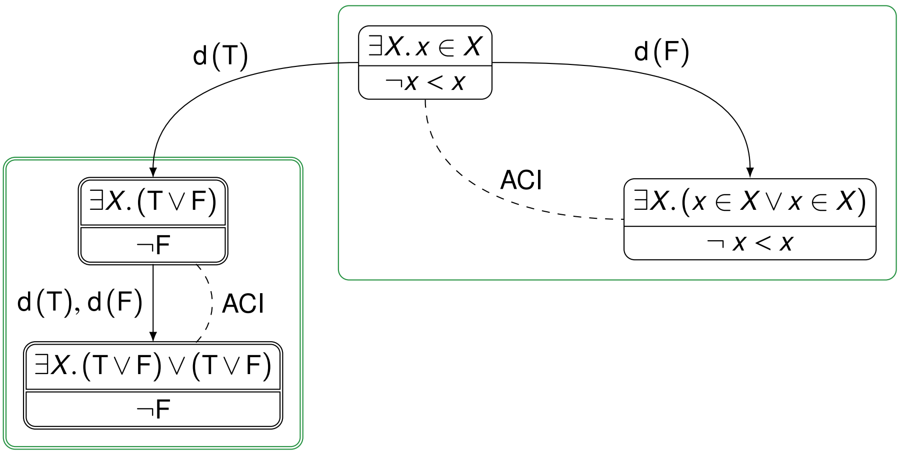
Let's define them on WS1S formulas directly!

$(\exists X.\, x \in X) \stackrel{?}{\equiv} (\neg x < x)$ for $\Sigma = \{(\mathsf{T}), (\mathsf{F})\}$



Derivative

$$
\begin{array}{lcl}
\mathsf{d}\ v\ \mathsf{T} & = & \mathsf{T} \\
\mathsf{d}\ v\ \mathsf{F} & = & \mathsf{F} \\
\mathsf{d}\ v\ (x \in X) & = & \begin{cases} x \in X & \text{if } \neg v[x] \\ \mathsf{T} & \text{if } v[x] \wedge v[X] \\ \mathsf{F} & \text{otherwise} \end{cases} \\
\mathsf{d}\ v\ (x < y) & = & \cdots \\
\mathsf{d}\ v\ (\varphi \vee \psi) & = & \mathsf{d}\ v\ \varphi \vee \mathsf{d}\ v\ \psi \\
\mathsf{d}\ v\ (\neg \varphi) & = & \neg\, \mathsf{d}\ v\ \varphi \\
\mathsf{d}\ v\ (\exists X.\, \varphi) & = & \exists X.\, (\mathsf{d}\ (v_{X \mapsto \mathsf{T}})\ \varphi \vee \mathsf{d}\ (v_{X \mapsto \mathsf{F}})\ \varphi)
\end{array}
$$

Acceptance Test

$$
\begin{aligned}
\varepsilon\,\mathsf{T} &= \mathsf{T}\\
\varepsilon\,\mathsf{F} &= \mathsf{F}\\
\varepsilon\,(x \in X) &= \mathsf{F}\\
\varepsilon\,(x < y) &= \mathsf{F}\\
\varepsilon\,(\varphi \vee \psi) &= \varepsilon\,\varphi \vee \varepsilon\,\psi\\
\varepsilon\,(\neg\varphi) &= \neg\,\varepsilon\,\varphi\\
\varepsilon\,(\exists X.\,\varphi) &= \text{action happens here}
\end{aligned}
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Acceptance Test

$$
\begin{aligned}
\varepsilon\,\mathsf{T} &= \mathsf{T} \\
\varepsilon\,\mathsf{F} &= \mathsf{F} \\
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\varepsilon\,(x < y) &= \mathsf{F} \\
\varepsilon\,(\varphi \vee \psi) &= \varepsilon\,\varphi \vee \varepsilon\,\psi \\
\varepsilon\,(\neg\varphi) &= \neg\,\varepsilon\,\varphi \\
\varepsilon\,(\exists X.\,\varphi) &= \text{action happens here}
\end{aligned}
$$

futurization

derivatives
from the right

Altogether

A decision procedure for WS1S that

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Thanks. Questions?

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