**Technische Universität München (I21)** **Summer term 2016**
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# Decision Procedures – Homework 2

Discussed on Tuesday, 28th July, 2016.

## Exercise 2.1 Proving equivalence of programs

Show that the two programs do the same, using uninterpreted functions

| (a) | (b) |
|---|---|
| int factorial3_a (int in) {<br>int i, out_a=in;<br>for (i=0; i<2; i++)<br>out_a=out_a*(in-i);<br>return out_a;<br>} | int factorial3_b (int in) {<br>int out_b;<br><br>out_b= in * (in-1) * (in-2)<br>return out_b<br>} |

**Solution:**
Outcome of the compiling Process

| | |
|---|---|
| $in1 = in - 1$<br>$in2 = in - 2$<br>$a = in * in1$<br>$out\_a = out1\_a * in2$ | $in1 = in - 1$<br>$in2 = in - 2$<br>$out\_b = in * in1 * in2$ |

Verification Condition

| | |
|---|---|
| $in1 = in - 1 \wedge$<br>$in2 = in - 2 \wedge$<br>$a = in * in1 \wedge$<br>$out\_a = out1\_a * in2$<br>$\Rightarrow out\_b = in * (in - 1) * (in - 2)$ | $in1 = in - 1 \wedge$<br>$in2 = in - 2 \wedge$<br>$out\_b = in * in1 * in2$<br>$\Rightarrow out\_b = in * (in - 1) * (in - 2)$ |

Abstracted Version

$out\_a = G(G(in, F(in, 1)), F(in, 2))$ | $out\_b = G(G(in, F(in, 1)), F(in, 2))$

## Exercise 2.2 Ackermann's Reduction

Apply Ackermann's Reduction to the following term:

$$x_1 = x_2 \wedge y_1 = y_2 \wedge F(x_1, F(x_1, G(F(x_1, y_1)))) = F(x_2, F(x_2, G(F(x_2, y_2))))$$

**Solution:**

- Indexing

| | |
|---|---|
| $f_1 := F(x_1, y_1)$ | $f_4 := F(x_2, y_2)$ |
| $g_1 := G(f_1)$ | $g_2 := G(f_4)$ |
| $f_2 := F(x1, g_1)$ | $f_5 := F(x2, g_2)$ |
| $f_3 := F(x1, f_2)$ | $f_6 := F(x2, f_5)$ |

- Transform flat

  $flat^E := x_1 = x_2 \Rightarrow f_3 = f_6$

- Formulate conjunction of constraints

$$
\begin{aligned}
FC^E := \quad & x_1 = x_2 \wedge y_1 = y_2 \Rightarrow f_1 = f_4 \wedge \\
& f_1 = f_4 \Rightarrow g_1 = g_2 \wedge \\
& x_1 = x_2 \wedge g_1 = g_2 \Rightarrow f_2 = f_5 \wedge \\
& x_1 = x_2 \wedge f_2 = f_5 \Rightarrow f_3 = f_6 \wedge \\
& g_1 = y_1 \Rightarrow f_2 = f_1 \wedge \\
& g_2 = y_2 \Rightarrow f_5 = f_4 \wedge \\
& f_2 = g_1 \Rightarrow f_3 = f_2 \wedge \\
& f_5 = g_2 \Rightarrow f_6 = f_5 \wedge \\
& f_2 = y_1 \Rightarrow f_3 = f_1 \wedge \\
& f_5 = y_2 \Rightarrow f_6 = f_4 \wedge \\
& x_1 = x_2 \wedge f_5 = y_1 \Rightarrow f_1 = f_6 \wedge \\
& x_1 = x_2 \wedge f_2 = y_2 \Rightarrow f_3 = f_4 \wedge \\
& x_1 = x_2 \wedge f_5 = g_1 \Rightarrow f_2 = f_6 \wedge \\
& x_1 = x_2 \wedge f_2 = g_2 \Rightarrow f_3 = f_5 \wedge \\
& x_1 = x_2 \wedge g_1 = y_2 \Rightarrow f_2 = f_4 \wedge \\
& x_1 = x_2 \wedge g_2 = y_1 \Rightarrow f_1 = f_5
\end{aligned}
$$

- $\varphi^E := FC^E \rightarrow flat^E$

## Exercise 2.3 Congruence Closure

Apply the congruence closure algorithm to the following terms:

(a) $\varphi_a^{UF} := x_1 = x_2 \wedge x_2 \neq x_3 \wedge x_3 = x_4 \wedge x_4 = x_5 \wedge F(x1) \neq F(x3)$

(b) $\varphi_b^{UF} := x_1 = x_2 \wedge x_2 = x_3 \wedge x_3 \neq x_4 \wedge F(x_1, x_2) = F(x_2, x_4)$

(c) $\varphi_c^{UF} := x_1 = x_2 \wedge x_2 \neq x_3 \wedge F(x_1) = F(x_2) \wedge \neg(F(x_1) \neq G(x_3) \vee G(x_2) = G(x_3))$

(d) $\varphi_d^{UF} := F(F(F(x))) = x \wedge F(F(F(F(F(x))))) = x \wedge F(x) \neq x$

**Solution:**

(a) $\{x_1, x_2\}\{x_3, x_4, x_5\}\{F(x_1)\}\{F(x_3)\}$
$\Rightarrow$Satisfiable

(b) $\{x_1, x_2, x_3\}\{x_4\}\{F(x_1, x_2), F(x_2, x_4)\}$
$\Rightarrow$Unsatisfiable

(c) $\{x_1, x_2\}\{x_3\}\{F(x_1), F(x_2), G(x_3)\}\{G(x_2)\}$
$\Rightarrow$Satisfiable

(d) $\{x, F(x), F(F(F(x))), F(F(F(F(F(x)))))\}$
$\Rightarrow$Unsatisfiable