

Exercise 1 (λ -Terms)

Rewrite the following terms such that they are completely parenthesized and conform to the grammar for the λ -calculus given in the lecture (without any shortcut notations).

a) $u x (y z) (\lambda v. v y)$

b) $(\lambda x y z. x z (y z)) u v w$

Rewrite the following terms such there are as few parentheses as possible, and apply all shortcut notation from the lecture:

c) $((u (\lambda x. (v (w x)))) x)$

d) $((((w (\lambda x. (\lambda y. (\lambda z. ((x z) (y z)))))) u) v)$

Evaluate the following substitutions:

e) $(\lambda y. x (\lambda x. x)) [(\lambda y. x y)/x]$

f) $(y (\lambda v. x v)) [(\lambda y. v y)/x]$

Solution

a) $((u x) (y z)) (\lambda v. (v y))$

b) $((((\lambda x. (\lambda y. (\lambda z. ((x z) (y z)))) u) v) w)$

c) $u (\lambda x. v (w x)) x$

d) $w (\lambda x y z. x z (y z)) u v$

e)

$$\begin{aligned} & (\lambda y. x (\lambda x. x)) [(\lambda y. x y)/x] \\ = & (\lambda y. x [(\lambda y. x y)/x] (\lambda x. x) [(\lambda y. x y)/x]) \\ = & (\lambda y. (\lambda y. x y) (\lambda x. x) [(\lambda y. x y)/x]) \\ = & (\lambda y. (\lambda y. x y) (\lambda x. x)) \end{aligned}$$

f)

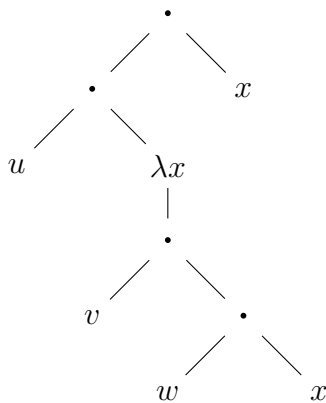
$$\begin{aligned}
 & (y (\lambda v. x v)) [(\lambda y. v y)/x] \\
 = & y [(\lambda y. v y)/x] (\lambda v. x v) [(\lambda y. v y)/x] \\
 = & y (\lambda v. x v) [(\lambda y. v y)/x] \\
 = & y (\lambda v'. (x v) [v'/v]) [(\lambda y. v y)/x] \\
 = & y (\lambda v'. (x [v'/v] v [v'/v])) [(\lambda y. v y)/x] \\
 = & y (\lambda v'. (x v [v'/v])) [(\lambda y. v y)/x] \\
 = & y (\lambda v'. (x v')) [(\lambda y. v y)/x] \\
 = & y (\lambda v'. x [(\lambda y. v y)/x] v' [(\lambda y. v y)/x]) \\
 = & y (\lambda v'. (\lambda y. v y) v' [(\lambda y. v y)/x]) \\
 = & y (\lambda v'. (\lambda y. v y) v')
 \end{aligned}$$

Exercise 2 (λ -Terms as Trees)

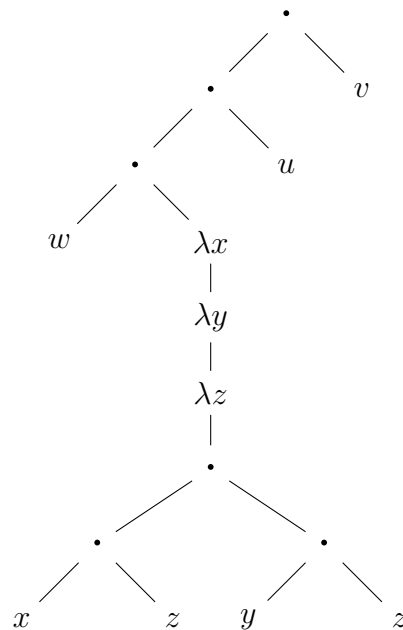
Rewrite the λ -terms resulting from exercises 1c) and 1d) to their corresponding representation as a tree.

Solution

a)



b)



Exercise 3 (Formalisations with λ -Terms)

Express the following propositions as λ -terms. Use the constant D as a derivative operator.

- The derivative of x^2 is $2x$.
- The derivative of x^2 at 3 is 6.

- c) Let f be a function, and let g be defined as $g(x) := f(x^2)$. The derivative of g at x is different from the derivative of f at x^2 .
- d) Formulate the proposition c) without using the auxiliary function symbol g .

Solution

- a) $(=) (D (\lambda x. (\wedge) x 2)) (\lambda x. (*) 2 x)$
- b) $(=) (D (\lambda x. (\wedge) x 2) 3) 6$
- c) $(\neq) (D g x) (D f ((\wedge) x 2))$
- d) $(\neq) (D (\lambda x. f ((\wedge) x 2)) x) (D f ((\wedge) x 2))$

Homework 4 (Interpreting λ -Terms)

Give a *compact* natural-language description of the computational effect of the following λ -terms.

- a) $\lambda x. x$
- b) $\lambda x y. x$
- c) $\lambda x y z. x z y$
- d) $\lambda x y. x (x y)$
- e) $\lambda x y z. x (y z)$

Homework 5 (Free and Bound Variables)

Mark the free variables in the following examples. Graphically indicate (by drawing arrows) the binding λ for each bound variable.

- a) $\lambda x y z. (\lambda x y. z x) y (x z)$
- b) $\lambda x. \lambda y. (\lambda y. z (\lambda z. y x)) (\lambda x z. x y z) y x$

Homework 6 (Substitutions)

Evaluate the following substitutions:

- a) $((\lambda x. f x) (\lambda f. f x)) [g x / f]$
- b) $(\lambda f. \lambda y. f x y) [f y / x]$

Homework 7 (Properties of Substitution)

Evaluate the following substitutions:

- a) Give a counterexample for

$$s[t/x][u/y] = s[u/y][t/x].$$

- b) Under which conditions does

$$s[t/x][u/y] = s[t[u/y]/x]$$

hold?