Homework 1 (Normal Forms)

Recall the inductive definition of the set NF of normal forms:

\[
\begin{align*}
  t \in \text{NF} & \land \forall x. t \in \text{NF} \\
  n \geq 0 & \land t_1 \in \text{NF} \land t_2 \in \text{NF} \land \ldots \land t_n \in \text{NF} \\
  x. t_1 t_2 \ldots t_n \in \text{NF}
\end{align*}
\]

Show that this set precisely captures all normal forms, i.e.:

\[
t \in \text{NF} \iff \exists t'. t \rightarrow^* \beta' t'
\]

Homework 2 (Weakly Normalising Terms)

Inductively define the set of weakly normalising terms WN, i.e. the set of terms that have a $\beta$-normal form. In particular it should hold that

\[
s \in \text{WN} \iff \exists t. s \Rightarrow_n t.
\]

Similarly, define the set of strongly normalising terms SN where a term $s$ is strongly normalising if there is no infinite sequence \( \{ t_i, i \in \mathbb{N} \} \) with $s \rightarrow^*_\beta t_0$ and $t_i \rightarrow^* \beta t_{i+1}$ for $i \in \mathbb{N}$.

Give a term $t$ that is weakly but not strongly normalising.