Exercise 1 (Progress Property)

Let $t$ be a closed and well-typed term, i.e. $\emptyset \vdash t : \tau$ for some $\tau$. Show that $t$ is either a value or there is a $t'$ such that $t \rightarrow_{cbv} t'$. 
Exercise 2 (Normal Form)
Show that every type-correct $\lambda^\gamma$-term has a $\beta$-normal form.
Homework 3 (Typing)

a) Prove:

\[
\vdash (\lambda x : \tau_2 \rightarrow \tau_3. \lambda y : \tau_1 \rightarrow \tau_2. \lambda z : \tau_1. \ x (y \ z)) : (\tau_2 \rightarrow \tau_3) \rightarrow (\tau_1 \rightarrow \tau_2) \rightarrow \tau_1 \rightarrow \tau_3
\]

b) Give suitable solutions for $?\tau_1$, $?\tau_2$, $?\tau_3$ and $?\tau_4$ and prove that the term is type-correct given your solution.

\[
\vdash \lambda x : ?\tau_1. \lambda y : ?\tau_2. \lambda z : ?\tau_3. \ x\ y\ (y\ z) : ?\tau_4
\]

Homework 4 ($\beta$-reduction preserves types)

A type system has the subject reduction property if evaluating an expression preserves its type. Prove that the simply typed $\lambda$-calculus ($\lambda^\tau$) has the subject reduction property:

\[
\Gamma \vdash t : \tau \land t \rightarrow_\beta t' \implies \Gamma \vdash t' : \tau
\]

Hints: Use induction over the inductive definition of $\rightarrow_\beta$ (Def. 1.2.2). State your inductive hypotheses precisely – it may help to introduce a binary predicate $P(t, t')$ to express the property you are proving by induction. Also note that the proof will require rule inversion: Given $\Gamma \vdash t : \tau$, the shape of $t$ (variable, application, or $\lambda$-abstraction) may determine which typing rule must have been used to derive the typing judgment.

Within your proof, you are free to use the following lemma about substitution:

\[
\Gamma \vdash u : \tau_0 \land \Gamma[x : \tau_0] \vdash t : \tau \implies \Gamma \vdash t[u/x] : \tau
\]

Homework 5 (Implementation of multiset-ordering and reduction)

Implement the multiset ordering and the reduction strategy from the second tutorial exercise in your favorite programming language.