Logic Exercises

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Here is a website for syntax trees and truth tables.

Exercise 1.1. [**Hello Logic**]
Discuss: What does logic mean to you? Is it worth studying? Why? Why not? Where do we use logic? How did it come into being? What makes logic special?

Exercise 1.2. [**Basics**]
Let M be a set of formulas, and let F and G be formulas. Which of the following assertions hold?

1. If F satisfiable then $M \models F$
2. F is valid iff $\top \models F$
3. If $\models F$ then $M \models F$
4. If $M \models F$ then $M \cup \{G\} \models F$
5. $M \models F$ and $M \models \neg F$ cannot hold simultaneously
6. If $M \models G \to F$ and $M \models G$ then $M \models F$

Exercise 1.3. [**Coincidence Lemma**]
Assume that for all atomic formulas A_i in F, $\mathcal{A}(A_i) = \mathcal{A}'(A_i)$. Show that

$$\mathcal{A} \models F \text{ iff } \mathcal{A}' \models F$$

Exercise 1.4. [**Anti-Interpolant**]
Assume F and G do not share any atoms. Show that if $\models F \to G$ then F is unsatisfiable or G is a tautology (or both). *Hint:* you may want to use the previous result.

Exercise 1.5. [**Sense and Reference**]
Pick an assignment $\mathcal{W}$. Call this assignment *the world*. Now pick a formula F suitable for $\mathcal{W}$. Then either $\mathcal{W} \models F \leftrightarrow \top$ or $\mathcal{W} \models F \leftrightarrow \bot$. Hence, each such formula F is equal to $\top$ or $\bot$ under $\mathcal{W}$.

Discuss: Do you agree? For example, should we treat $F \vee \neg F$ as being equal to $\top$? Do both hold the same cognitive value?

Homework: Homework exercises will not be graded. Rather, you can ask for help and discuss the exercises and your solutions on Zulip.

Homework 1.1. [CNF and DNF] (+)
Use the rewriting-based procedure from the lecture to convert the following formulas F and G first to NNF, and then to CNF and DNF. Document each rewriting step.

$$F = \neg\neg(\neg A_1 \wedge \neg\neg(A_2 \vee A_3)) \qquad G = (A_1 \vee A_2 \vee A_3) \wedge (\neg A_1 \vee \neg A_2)$$

Homework 1.2. [Basic equivalences] (+)
Let F and G be formulas. Are the following statements equivalent? Proof or counterexample!

1. $\models F \leftrightarrow G$
2. $F \equiv G$

What is the difference between $F \leftrightarrow G$ and $F \equiv G$?
How about these two statements? Prove or disprove!

1. F is valid
2. $F \equiv \top$

Homework 1.3. [Efficient CNF satisfiability check] (++)
In general, solving satisfiability for CNF formula is a hard problem. Consider the special case where clauses may only contain up to two literals. Give a polynomial time algorithm to check for satisfiability.

Homework 1.4. [Craig-Interpolant] (+++)
Let F and G be arbitrary formulas with $F \models G$. Show that there is a formula H mentioning only propositional variables occuring in both F and G such that $F \models H$ and $H \models G$.

> There can be no doubt that the knowledge of logic is of considerable practical importance for everyone who desires to think and infer correctly.
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> — Alfred Tarski