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Equational Logic
Summer Term 2012
Exercise Sheet 8
June 15

Homework is due on June 22nd, before the tutorial.

Exercise 1 (H) (*Polynomial Interpretation*)

Use the polynomial interpretation $\mathcal{A}$ with $A = \mathbb{N} - \{0, 1, 2\}$ and $P_f(X, Y) = X^2 + XY$ to show that the following term rewriting system terminates:

$$\{\ f(f(x, y), z) \longrightarrow f(x, f(y, z)),\ f(x, f(y, z)) \longrightarrow f(y, y)\ \}$$

Exercise 2 (H) (*Critical Pairs*)

Specify all critical pairs of the following term rewriting systems:

a) $f(g(f(x))) \longrightarrow x,\ f(g(x)) \longrightarrow g(f(x))$

b) $g(f(x)) \longrightarrow f(g^2(h(x))),\ h(f(x)) \longrightarrow f(g(h^2(x))),\ h(g(x)) \longrightarrow g(h(x))$

c) $f(x, x) \longrightarrow a,\ f(x, g(x)) \longrightarrow b$

d) $f(f(x, y), z) \longrightarrow f(x, f(y, z)),\ f(x, a) \longrightarrow x$

e) $f(f(x, y), z) \longrightarrow f(x, f(y, z)),\ f(a, x) \longrightarrow x$

f) $0 + y \longrightarrow y,\ x + 0 \longrightarrow x,\ s(x) + y \longrightarrow s(x + y),\ x + s(y) \longrightarrow s(x + y)$

Which systems are locally confluent, which are convergent (i.e., terminating and confluent)
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Exercise 3 (H) (*Diamond Property*)

Let $\longrightarrow$ be a TRS with the following „diamond property":

$$y \longleftarrow x \longrightarrow z \ \wedge\ y \neq z \quad\Longrightarrow\quad \exists u.\ y \longrightarrow u \longleftarrow z$$

Show that, if a has a normal form, all recuctions from a to this normal form have the same length.

Exercise 4 (T) (*Polynomial Interpretation*)

Specify a polynomial interpretation that demonstrates termination of the following term rewriting system:

$$\{(x \oplus y) \oplus z \longrightarrow x \oplus (y \oplus z),\ x \odot (y \oplus z) \longrightarrow (x \odot y) \oplus (x \odot z)\}$$

Exercise 5 (T) (*Newman's Lemma*)

Give an indirect proof of Newman's Lemma, by showing that $\longrightarrow$ has an infinite reduction sequence, if $\longrightarrow$ is locally confluent but not confluent.

Hint: Show that every element with two distinct normal forms has a direct successor with two distinct normal forms.

Exercise 6 (T) (*Confluence*)

Determine terms r_1 and r_2 such that $\{f(g(x)) \longrightarrow r_1,\ g(h(x)) \longrightarrow r_2\}$ is confluent.