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Equational Logic
Summer Term 2012
Exercise Sheet 9
June 22

Homework is due on June 29th, before the tutorial.

Exercise 1 (H) (*Completion*)

Complete

$$\{f(g(f(x))) \approx x\}$$

to a convergent term rewriting system!

Exercise 2 (H) (*Linear Term Rewriting Systems*)

A rewrite rule $l \longrightarrow r$ is called *left-linear* if every variable in l occurs exactly once. Similarly, $l \longrightarrow r$ is called *right-linear* if every variable in r occurs exactly once. A rule is *linear* if it is both right- and left-linear. We say that a term rewriting system is *linear* if it contains only linear rules.

Show:

a) Every linear term rewriting system R that has no critical pairs is confluent. (You must give a self-contained proof; do not simply apply Corollary 6.3.11 from the textbook! *Hint:* Show that R is strongly confluent.)

b) If R is a linear term rewriting system, and for every critical pair (t_1, t_2) there exists t_0 such that $t_1 \stackrel{=}{\longrightarrow} t_0 \stackrel{=}{\longleftarrow} t_2$, then R is confluent. *Hint:* Extend your proof of a).

Exercise 3 (T) (*Confluence*)

Let R be the following term rewriting system:

$$\{f(x,x) \longrightarrow a,\ c \longrightarrow g(c),\ g(x) \longrightarrow f(x, g(x))\}$$

Is R confluent? Justify your answer.

Exercise 4 (T) (*Completion*)

Complete

$$\{f(g(f(x))) \approx f(g(x))\}$$

to a convergent term rewriting system!

Exercise 5 (T) (*Completion*)

You are given the following equations:

$$E = \{1 \cdot x \approx x,\ x \cdot 1 \approx x,\ i(x) \cdot (x \cdot y) \approx y\}$$

Construct a term rewriting system that decides equality with respect to E.