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Equational Logic
Summer Term 2015
Exercise Sheet 2

Exercise 8 (Termination)

Show that the following programs, with variables over the natural numbers, terminate on all valid inputs.

a) ggT(m, n)

> **while** $m \neq n$ **do**
>   **if** $m > n$ **then** $m := m - n$ **else** $n := n - m$

b) ggT(m, n)

> **while** $m \neq n$ **do**
>   **if** $m > n$ **then** $m := m - n$
>   **else begin** $h := m;\ m := n;\ n := h$ **end**

c) The function f, which is defined recursively as follows.

$$\begin{aligned}
f(m, n, 0) &= m + n \\
f(m, 0, k+1) &= f(m + k, 1, k) \\
f(m, n+1, k+1) &= f(m + k, f(m + k, n, k + 1), k)
\end{aligned}$$

Exercise 9 (Product Order)

Show that the lexicographic product $(A \times B, >_{A\times B})$ of the orders $(A, >_A)$ and $(B, >_B)$ is a total order, if $>_A$ and $>_B$ are total orders.

Exercise 10 (Positions)

Let s, t be terms, and Let $p, q \in \mathrm{Pos}(s)$ be parallel positions in s (i.e., $p \parallel q$). Show $(s[t]_p)|_q = s|_q$.

Homework 11 (Positions')

Prove the following properties using induction on the length of words denoting positions.

If $p \in \mathrm{Pos}(s)$ and $q \in \mathrm{Pos}(t)$, then

a) $(s[t]_p)|_{pq} = t|_q$

b) $(s[t]_p)[r]_{pq} = s[t[r]_q]_p$

Homework 12 (Playing with the devil)

The devil made up the following game to torture the souls of poor students:

In a box, there is an unknown number of red, green and blue balls. The player has to draw one ball from the box, without seeing its colour beforehand. The drawn ball is removed from the box, but depending on its colour, one of these things happens:

- If it is a red ball, the devil adds an arbitrary number of green balls to the box.
- If it is a green ball, the devil adds an arbitrary number of blue balls to the box.
- If it is a blue ball, nothing happens.

The game ends when the box is empty.

Given any box, what is the probability that the player can end the game in finitely many steps?

Homework 13 (Multiset ordering)

a) Given a strict order $(A, >)$, define the following single-step relation on $\mathcal{M}(A)$:

$$M >^1_{mul} N :\Leftrightarrow \exists x \in M, Y \in \mathcal{M}(A).\ N = (M - \{x\}) \cup Y \wedge \forall y \in Y.\ x > y$$

Show that the transitive closure of $>^1_{mul}$ is contained in $>_{mul}$.

b) Show that if the ordering $>$ is total, then its multiset extension $>_{mul}$ is total.