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Equational Logic
Summer Term 2015
Exercise Sheet 6

Exercise 29 (Most General Unifier)

Let S and T be unification problems. Moreover, let σ be a most general unifier for S and θ be a most general unifier for $\sigma(T)$. Show that $\theta\sigma$ is a most general unifier for $S \cup T$.

Exercise 30 (Equivalence Relations)

Prove that the notions of a *equivalence relation* and a *partition* coincide.

Reminder: A set of sets P is called a partition of a set M iff all elements of P are pairwise disjoint, P does not contain the empty set, and $\bigcup P = M$.

Exercise 31 (Termination)

A term rewriting system R is called *right reduced*, if for all $(l \longrightarrow r) \in R$, the right hand side r is irreducible. Show that every right reduced and right ground term rewriting system terminates.

Hint: Consider the positions in the term at which rules from R may be applied, and specify a suitable order on terms. Is there a simpler way to get this lemma as a corollary from a lemma that was presented in the lecture?

Homework 32 (Compactness)

Prove that every satisfiable set of equations over a finite set of variables contains a finite subset that has the same solutions.

Note that equations are interpreted in the term algebra.

Hint: Select a countable subset of the set of equations.

Homework 33 (Deciding Termination for Right-Ground TRSs)

In the lecture, we discussed a procedure to decide termination of right-ground term rewriting systems. It is important that we use a breadth-first search strategy, as you shall demonstrate in this exercise.

a) Given the following procedure that uses a depth-first approach:

   Input A right ground term rewriting system $R = \{l_1 \longrightarrow r_1, \dots, l_n \longrightarrow r_n\}$.

Procedure Enumerate all reduction sequences that start with r_1, in depth-first order. If one of those sequences contains r_1 as a subterm, output *non-terminating*. Otherwise continue with the sequences starting at r_2, and so on. If all right hand sides have been processed, output *terminating*.

Find a right-ground term rewriting system such that the above procedure does not terminate.

b) Determine whether the following rewriting systems terminate using the breadth-first approach:

$$R_1 = \{f(x, x) \longrightarrow f(a, b), b \longrightarrow c\}$$
$$R_2 = \{f(x, x) \longrightarrow f(a, b), b \longrightarrow a, b \longrightarrow c\}$$

c) Implement the correct algorithm in Haskell. More instructions can be found on the lecture website.