Logics Exercise

TU München
Institut für Informatik

Prof. Tobias Nipkow
Lars Hupel

SS 2017 — Exercise Sheet 8 — 20.06.2017

Submission of homework: Before tutorial on 27.06.2017. You have to do the homework yourself; no teamwork allowed.

Exercise 8.1. [**Decidability**]

1. Resolution for first-order logic is sound and complete.
2. Satisfiability and validity for first-order logic are undecidable.

How do you reconcile these two facts?

Exercise 8.2. [**Ground Resolution**]
Use ground resolution to prove that the following formula is valid:

$$(\forall x P(x, f(x))) \longrightarrow \exists y P(c, y)$$

Exercise 8.3. [**Barber Paradox**]
Consider the following facts:

1. Every barber shaves those who do not shave themselves.
2. No barber shaves anyone who shaves themself.

Show with resolution that there are no barbers by resolution.

Homework 8.1. [Restricted Resolution] (8 points)
In the resolution procedure as defined in the lecture slides, we can unify arbitrarily many literals from two clauses. Consider a modified resolution procedure, where exactly one literal is picked. We add another rule ("collapsing rule"): For a clause $C = \{L_1, \ldots, L_n\}$, where $\{L_i, L_j\}$ can be unified using a mgu δ, add another clause $C' = (C - L_i)\delta$.

For example, given the clause

$$C = \{\neg W(x), \neg W(f(y)), T(x, y), \neg W(f(c))\}$$

we can apply the collapsing rule as follows:

$$L_1 = \neg W(x), L_2 = \neg W(f(y)), \delta = \{x \mapsto f(y)\}, C' = \{\neg W(f(y)), T(f(y), y), \neg W(f(c))\}$$

(Note that there are multiple possible ways to apply the collapsing rule to C.)

Prove that our modified resolution calculus, including collapsing rule, can be simulated by the original resolution calculus, and vice versa.

Homework 8.2. [Resolution] (8 points)
Show with resolution that:

1. $\forall x(\neg R(x) \longrightarrow R(f(x))) \longrightarrow \exists x(R(x) \wedge R(f(f(x))))$ is valid
2. $\exists x(P(x) \wedge \neg P(f(f(x)))) \wedge \forall x(P(x) \longrightarrow P(f(x)))$ is unsatisfiable

Homework 8.3. [Equality] (4 points)
We consider how to model equality in predicate logic. In the lecture slides, the following axiom schema for congruence is used:

$$\frac{Eq(x_i, y)}{Eq(f(x_1, \ldots, x_i, \ldots, x_n), f(x_1, \ldots, y, \ldots, x_n))}$$

Assume that this schema is replaced by:

$$\frac{Eq(x_1, y_1) \qquad \cdots \qquad Eq(x_n, y_n)}{Eq(f(x_1, \ldots, x_n), f(y_1, \ldots, y_n))}$$

Reflexivity, symmetry and transitivity stay unchanged. Show that the above modified schemas is equivalent to the schemas from the lecture.

Hint: Simulate the modified schema with the original one and vice versa.