Semantics of Programming Languages

Exercise Sheet 10

Exercise 10.1 Security type system: bottom-up with subsumption

Use the template file Ex10_Template.thy.

Recall security type systems for information flow control from the lecture. Such a type systems can either be defined in a top-down or in a bottom-up manner. Independently of this choice, the type system may or may not contain a subsumption rule (also called anti-monotonicity in the lecture). The lecture discussed already all but one combination: a bottom-up type system with subsumption.

- (a) Define a bottom-up security type system for information flow control with subsumption rule.
- (b) Prove the equivalence of the newly introduced bottom-up type system with the bottom-up type system without subsumption rule from the lecture.

Homework 10

Submission until Wednesday, January 26, 2011, 12:00 (noon). The first part (the definition) is already due on January 19.

Use the template file Dependency_Template.thy.

The task is to define a dependency analysis between variables. We say that variable x depends on y after command c if the value of y at the beginning of the execution of c may influence the value of x at the end of the execution.

For example, consider the program y ::= 0; IF $x \leq 2$ THEN y ::= x ELSE z ::= 0.

Here, the variable x depends only on itself, since it is never assigned.

The variable y clearly depends on x. It does not depend on itself, since it is initially assigned a constant value, hence the original value is irrelevant.

The variable z depends on itself, since it may keep its value, but it also depends on x, since the assignment to it occurs under a conditional depending on x.

In the program WHILE b DO (x := y; y := z) the variable x depends on both y and z (the value of z reaches x in the second iteration of the loop).

(a) Define an inductive relation *influences* :: $name \Rightarrow com \Rightarrow name \Rightarrow bool$ which specifies a dependency analysis.

- (b) Prove its soundness w.r.t. to the big-step semantics. That is, prove the lemma
- $\mathbf{lemma} \ deps_sound:$

 $"[[(c, s) \Rightarrow t; s = s' \text{ on deps } c x; (c, s') \Rightarrow t'] \\ \implies t x = t' x"$

where deps c x abbreviates $\{y. influences y c x\}$.