

# Concrete Semantics

with Isabelle/HOL

Tobias Nipkow

Fakultät für Informatik  
Technische Universität München

2021-10-6

# Chapter 1

## Introduction

① Background

② This Course

① Background

② This Course

# Why Semantics?

Without semantics,  
we do not really know what our programs mean.

We merely have a good intuition and a warm feeling.

Like the state of mathematics in the 19th century  
— before set theory and logic entered the scene.

# Intuition is important!

- You need a good intuition to get your work done efficiently.
- To understand the average accounting program, intuition suffices.
- To write a bug-free accounting program may require more than intuition!
- I assume you have the necessary intuition.
- This course is about “beyond intuition”.

# Intuition is not sufficient!

Writing **correct** language processors (e.g. compilers, refactoring tools, ...) requires

- a deep understanding of language semantics,
- the ability to *reason* (= perform proofs) about the language and your processor.

Example:

What does the correctness of a type checker even mean?  
How is it proved?

# Why Semantics??

We have a compiler — that is the ultimate semantics!!

- A compiler gives each individual program a semantics.
- It does not help with reasoning about the PL or individual programs.
- Because compilers are far too complicated.
- They provide the worst possible semantics.
- Moreover: compilers may differ!

# The sad facts of life

- Most languages have one or more compilers.
- Most compilers have bugs.
- Few languages have a (separate, abstract) semantics.
- If they do, it will be informal (English).

# Bugs

- Google “compiler bug”
- Google “hostile applet”  
Early versions of Java had various security holes. Some of them had to do with an incorrect *bytecode verifier*.

GI Dissertationspreis 2003:

Gerwin Klein: *Verified Java Bytecode Verification*

# Standard ML (SML)

First real language with a mathematical semantics:

Milner, Tofte, Harper:

The Definition of Standard ML. 1990.



Robin Milner (1934–2010)  
Turing Award 1991.

Main achievements: LCF (theorem proving)  
SML (functional programming)  
CCS,  $\pi$  (concurrency)

# The sad fact of life

SML semantics hardly used:

- too difficult to read to answer simple questions quickly
- too much detail to allow reliable informal proof
- not processable beyond  $\text{\LaTeX}$ , not even executable

## More sad facts of life

- Real programming languages *are* complex.
- Even if designed by academics, not industry.
- Complex designs are error-prone.
- Informal mathematical proofs of complex designs are also error-prone.

# The solution

## Machine-checked language semantics and proofs

- Semantics at least type-correct
- Maybe executable
- *Proofs machine-checked*

The tool:

Proof Assistant (PA)

or

Interactive Theorem Prover (ITP)

# Proof Assistants

- You give the structure of the proof
- The PA checks the correctness of each step
- Can prove hard and huge theorems

Government health warnings:

Time consuming  
Potentially addictive  
Undermines your naive trust in informal proofs

# Terminology

This lecture course:

Formal = machine-checked

Verification = formal correctness proof

Traditionally:

Formal = mathematical

# Two landmark verifications

C compiler

Competitive with gcc -O1



Xavier Leroy  
INRIA Paris  
using Coq

Operating system  
microkernel (L4)



Gerwin Klein (& Co)  
NICTA Sydney  
using Isabelle

# A happy fact of life

Programming language researchers  
are increasingly using PAs

# Why verification pays off

Short term: *The software works!*

Long term:

Tracking effects of changes by rerunning proofs

Incremental changes of the software  
typically require only incremental changes of the proofs

Long term much more important than short term:

**Software Never Dies**

① Background

② This Course

# What this course is *not* about

- Hot or trendy PLs
- Comparison of PLs or PL paradigms
- Compilers (although they will be one application)

# What this course *is* about

- Techniques for the description and analysis of
  - PLs
  - PL tools
  - Programs
- Description techniques: *operational semantics*
- Proof techniques: *inductions*

Both informally and formally (PA!)

# Our PA: Isabelle/HOL

- Started 1986 by Paulson (U of Cambridge)
- Later development mainly by Nipkow & Co (TUM) and Wenzel
- The logic HOL is ordinary mathematics

Learning to use Isabelle/HOL  
is an integral part of the course

All exercises require the use of Isabelle/HOL

# Why I am so passionate about the PA part

- It is the future
- It is the only way to deal with complex languages  
*reliably*
- I want students to learn how to write correct proofs
- I have seen too many proofs that look more like  
LSD trips than coherent mathematical arguments

# Overview of course

- Introduction to Isabelle/HOL
- IMP (assignment and while loops) and its semantics
- A compiler for IMP
- Hoare logic for IMP
- Type systems for IMP
- Program analysis for IMP

The semantics part of the course is mostly traditional

The use of a PA is leading edge

A growing number of universities offer related course

What you learn in this course goes far beyond PLs

It has applications in compilers, security,  
software engineering etc.

It is a new approach to informatics