Rule induction for Com

$$\forall c, \sigma, \sigma'.\ \langle c, \sigma\rangle \to \sigma' \Rightarrow P(c, \sigma, \sigma')$$

$$\Longleftrightarrow$$

$$\forall \sigma.\ P(\mathbf{skip}, \sigma, \sigma)$$

$$\forall x, a, m, \sigma.\ \langle a, \sigma\rangle \to m \Rightarrow P(x{:=}a, \sigma, \sigma[m/x])$$

$$\forall c_1, c_2, \sigma_0, \sigma_1, \sigma_2.$$
$$\left(\begin{array}{c} \langle c_1, \sigma_0\rangle \to \sigma_1 \wedge \langle c_2, \sigma_1\rangle \to \sigma_2 \\ P(c_1, \sigma_0, \sigma_1) \wedge P(c_2, \sigma_1, \sigma_2) \end{array}\right)$$
$$\Rightarrow P(c_1{;}c_2, \sigma_0, \sigma_2)$$

$$\forall c_1, c_2, \sigma_0, \sigma_1.$$
$$(\langle b, \sigma_0\rangle \to \mathbf{true} \wedge \langle c_1, \sigma_0\rangle \to \sigma_1 \wedge P(c_1, \sigma_0, \sigma_1))$$
$$\Rightarrow P(\mathbf{if}\ b\ \mathbf{then}\ c_1\ \mathbf{else}\ c_2, \sigma_0, \sigma_1)$$

$$\forall b, c, \sigma.\ \langle b, \sigma\rangle \to \mathbf{false} \Rightarrow P(w, \sigma, \sigma)$$

$$\forall b, c, \sigma_0, \sigma_1, \sigma_2.$$
$$\left(\begin{array}{c} \langle b, \sigma_0\rangle \to \mathbf{true} \wedge \langle c, \sigma_0\rangle \to \sigma_1 \wedge \langle w, \sigma_1\rangle \to \sigma_2 \\ P(c, \sigma_0, \sigma_1) \wedge P(w, \sigma_1, \sigma_2) \end{array}\right)$$
$$\Rightarrow P(w, \sigma_0, \sigma_2)$$

where $w = \mathbf{while}\ b\ \mathbf{do}\ c$