The Isabelle Refinement Framework

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- This talk: towards faster verified algorithms at manageable effort

Introduction

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procedure AUGMENT(g, f, p)
    $c_p \leftarrow \min\{g_f(u, v) \mid (u, v) \in p\}$
    for all $(u, v) \in p$ **do**
        if $(u, v) \in g$ **then** $f(u, v) \leftarrow f(u, v) + c_p$
        else $f(v, u) \leftarrow f(v, u) - c_p$
    return f

procedure EDMONDS-KARP(g, s, t)
    $f \leftarrow \lambda(u, v).\ 0$
    while exists augmenting path in g_f **do**
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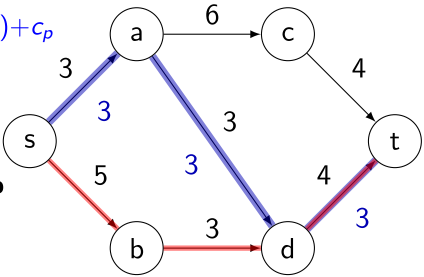

g: flow network s, t: source, target g_f: residual network

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Theorem (Ford-Fulkerson)

For a flow network g and flow f, the following 3 statements are equivalent

1. f *is a maximum flow*
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a few pages of definitions and textbook proof (e.g. Cormen).

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using basic concepts such as numbers, sets, and graphs. □

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int edmonds_karp(int s, int t) {
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   while (new_flow = bfs(s, t, parent)) {
      flow += new_flow;
      int cur = t;
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- Implementations used for different parts must fit together!

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shortest-path-spec

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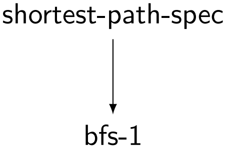

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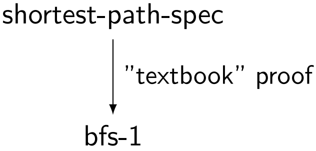

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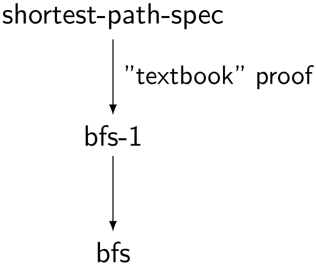

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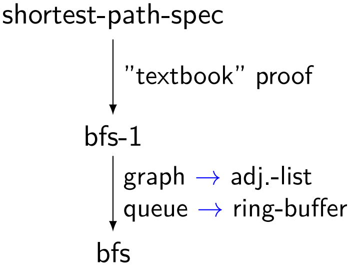

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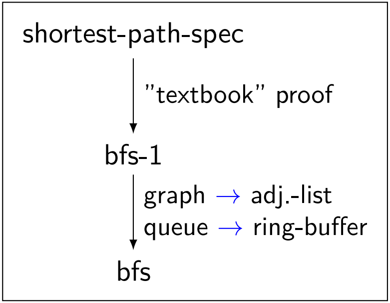

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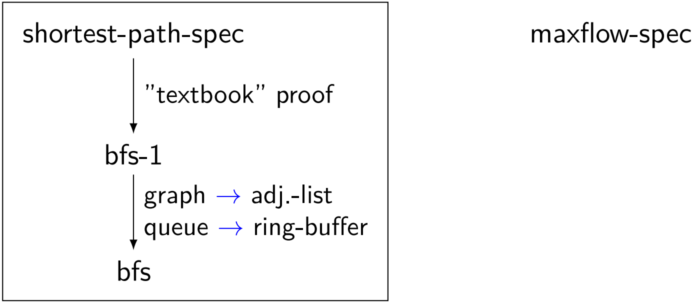

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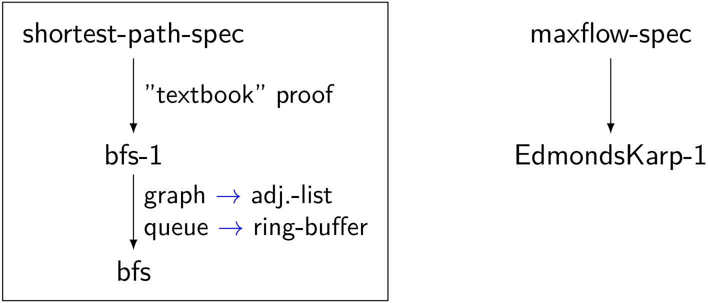

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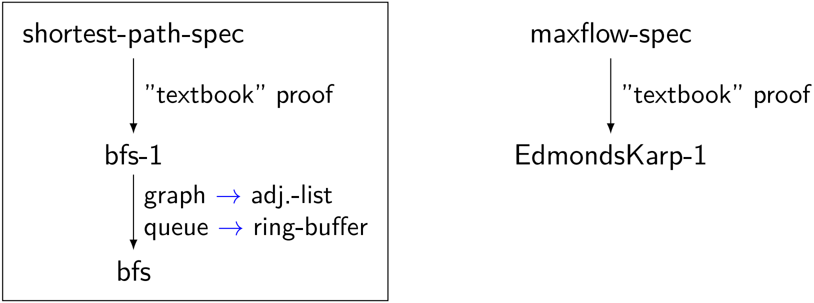

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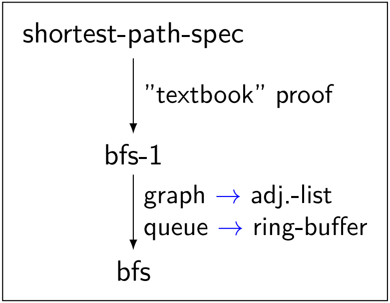

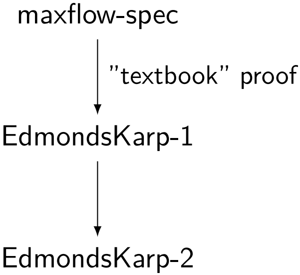

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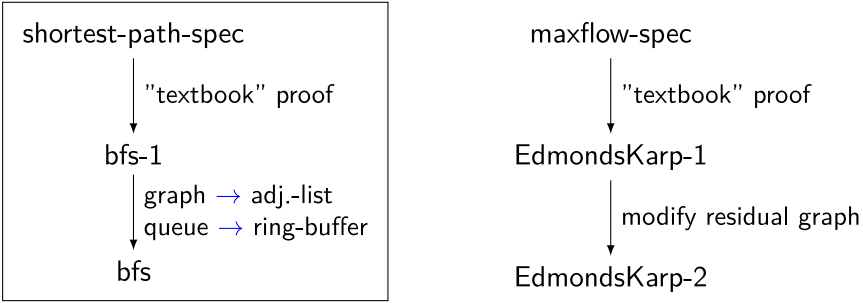

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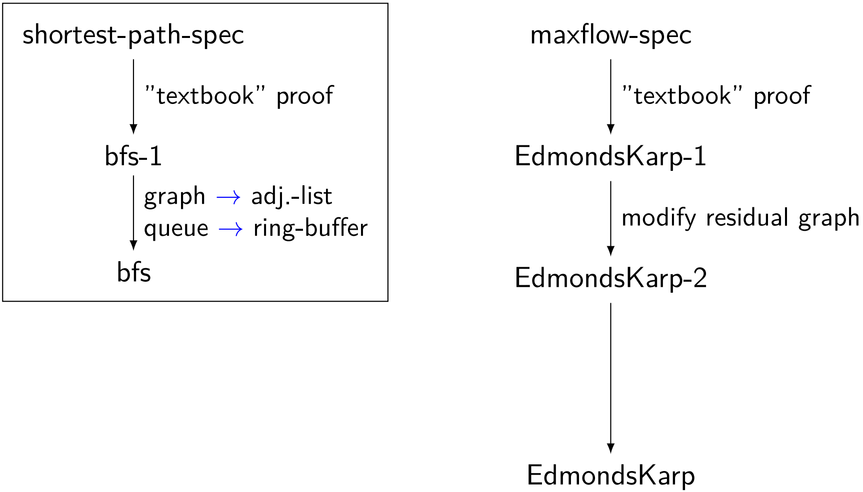

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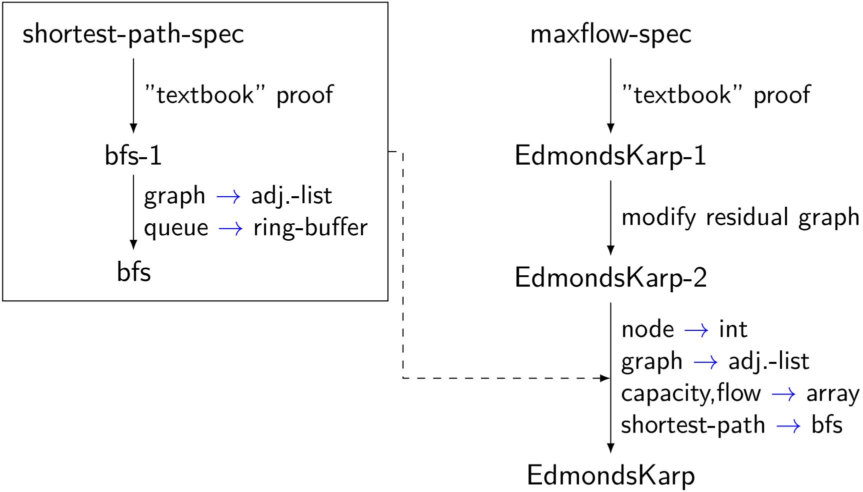

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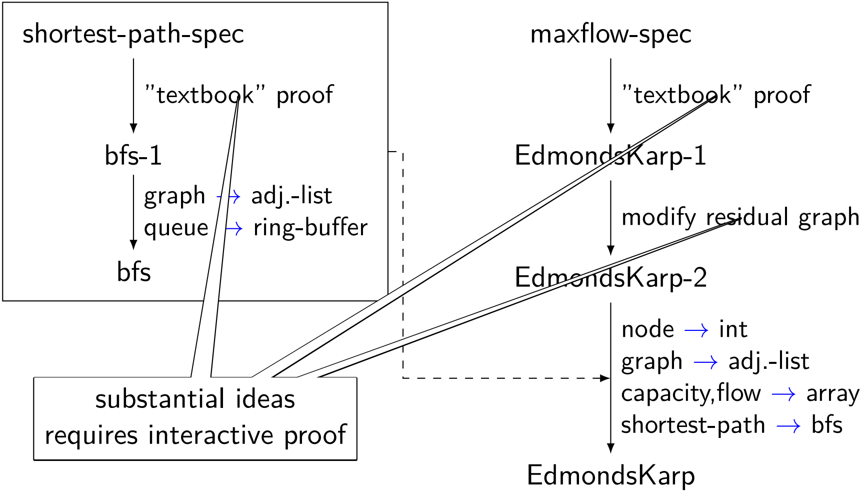

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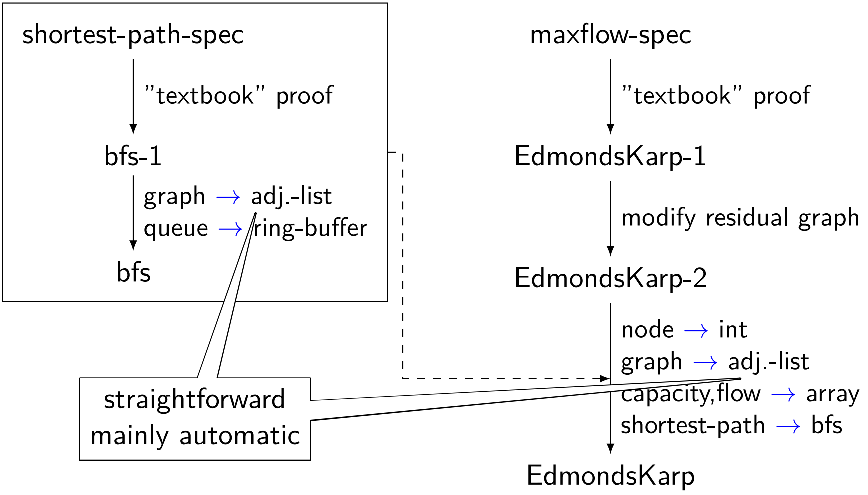

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- Automated transition from NRES to HEAP
  - automatic data refinement (e.g. integer by int64)
  - automatic placement on heap (e.g. list by array)
  - some in-bound proof obligations left to user

Code Generation

Translate HEAP to compilable code

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1. Imperative-HOL:
   - based on Isabelle's code generator
   - OCaml,SML,Haskell,Scala (using imp. features)
   - results cannot compete with optimized C/C++

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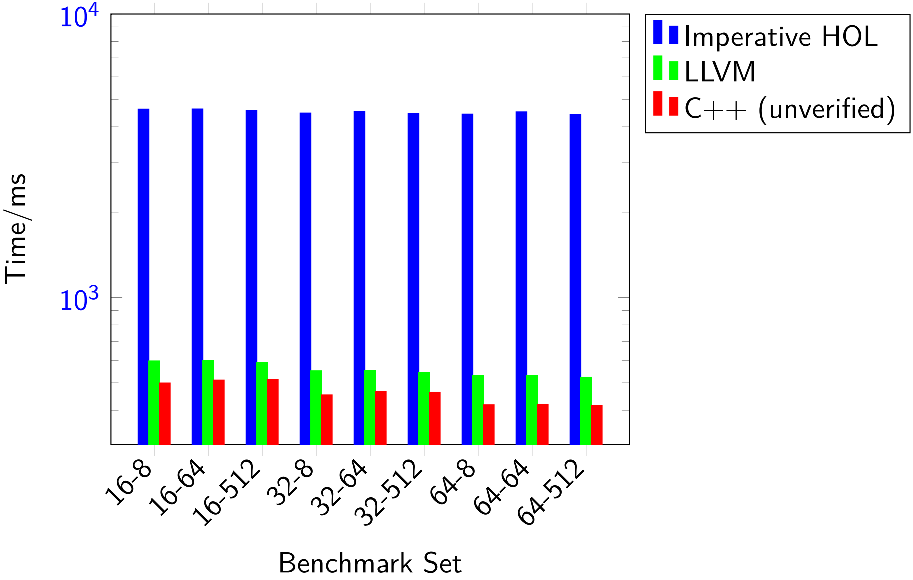
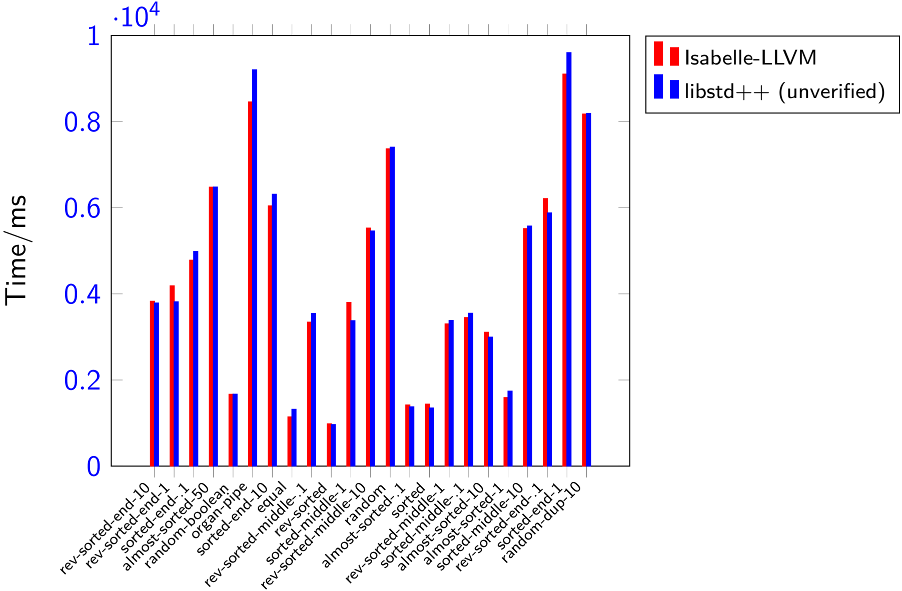
1. Imperative-HOL:
   - based on Isabelle's code generator
   - OCaml,SML,Haskell,Scala (using imp. features)
   - results cannot compete with optimized C/C++
2. NEW!: Isabelle-LLVM
   - shallow embedding of fragment of LLVM-IR
   - pretty-print to actual LLVM IR text
   - then use LLVM optimizer and compiler
   - faster programs
   - thinner (unverified) compilation layer

Knuth Morris Pratt



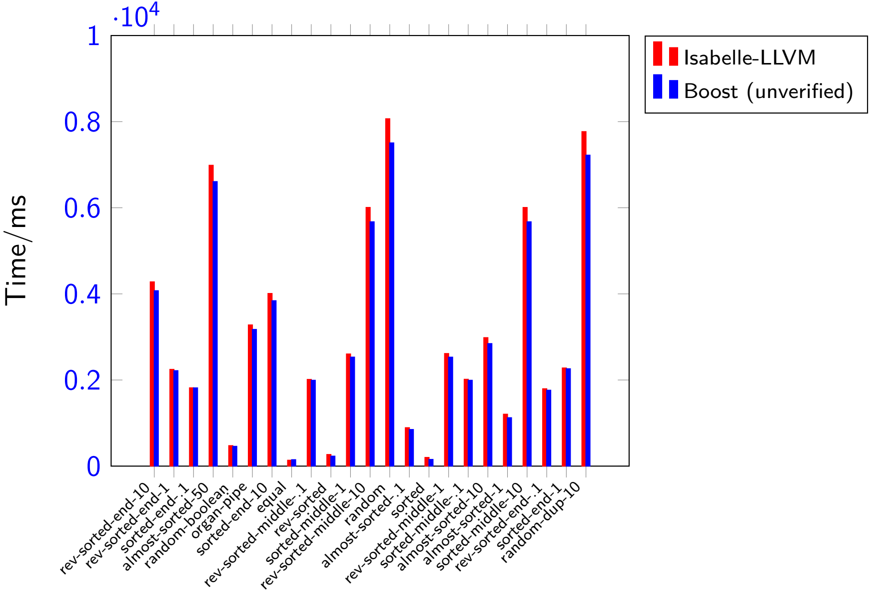
Execute *a*-*l* benchmark set from StringBench. Stop at first match.

Verified Sorting Algorithms: Introsort



Sorting $100 \cdot 10^6$ `uint64s` on Intel Core i7-8665U CPU, 32GiB RAM

Verified Sorting Algorithms: Pdqsort



Sorting $100 \cdot 10^6$ `uint64s` on Intel Core i7-8665U CPU, 32GiB RAM

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    - Scalable Sepref Tool

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```
f (l :: int list) {
 (int)set S = {}
 int c=0
 for (int i=0; i<|l|; ++i) {
  t1 = l[i]
  if (t1 ∉ S) {
   *: assert (c<|l|)
   ++c
   S={t1} ∪ S
  } } }
```

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```
f (l' :: int64 array) {
  hashmap S' = hm_empty()
  int64 c'=0
  for (int64 i'=0; i'<|l'|; ++i') {
   t1' = l[i']
   if (¬hm_member t1' S') {
    *:
    ++c'
    S'=hm_insert t1' S'
   } } free S' }
```

Sepref Tool

- Synthesize imperative program from functional
  - sep-logic assertion relating concrete with abstract variables

```
f (l :: int list) {
  (int)set S = {}
  int c=0
  for (int i=0; i<|l|; ++i) {
    t₁ = l[i]
    if (t₁ ∉ S) {
      *: assert (c<|l|)
      ++c
      S={t₁} ∪ S
    } } }
```

```
f (l′ :: int64 array) {
  hashmap S′ = hm_empty()
  int64 c′=0
  for (int64 i′=0; i′<|l′|; ++i′) {
    t₁′ = l[i′]
    if (¬hm_member t₁′ S′) {
      *:
      ++c′
      S′=hm_insert t₁′ S′
    } } free S′ }
```

At *: *array i64 l l′ * hm i64 S S′ * ...*

Nested Containers

Hoare-Rule for array-index:

$\{$array A l l$' *$ i64 i i$' *$ i$<|$l$|\}$ r$'$=l$'$[i$'$] $\{$ array A l l$' *$ i64 i i$' *$ A (l[i]) r$'$ $\}$

where

array A l p $= \exists$ l$'$. p+0 $\mapsto$ l$'$[0] $* \ldots *$ p+n $\mapsto$ l$'$[n]
$*$ A l[0] l$'$[0] $* \ldots *$ A l[n] l$'$[n]

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Problem: Does not work for *array* (*array i64*)! (result is shared)

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where

$$\text{array A l p} = \exists\ \text{l}'.\ \text{p}+0 \mapsto \text{l}'[0] * \ldots * \text{p}+\text{n} \mapsto \text{l}'[\text{n}]$$
$$* \text{ A l[0] l}'[0] * \ldots * \text{A l[n] l}'[\text{n}]$$

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  - None: element not in array
  - Manual ownership management

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where

$\text{array A l p} = \exists\ \text{l}'.\ \text{p}+0 \mapsto \text{l}'[0] * \ldots * \text{p}+\text{n} \mapsto \text{l}'[\text{n}]$
$\quad * \text{ A l[0] l}'[0] * \ldots * \text{A l[n] l}'[\text{n}]$

Problem: Does not work for *array* (*array i64*)! (result is shared)

- current approach: abstract data type: α *option list*
  - None: element not in array
  - Manual ownership management
- future:
  - read-only sharing (fractional sep-logic?)
  - automation (as far as possible)
  - maybe inspiration from Rust.

Conclusions

Isabelle Refinement Framework

  powerful interactive theorem prover
- $+$ stepwise refinement
- $+$ libraries for standard DS and algorithms
- $+$ lot's of automation
- $+$ efficient backend (LLVM)
- $=$ verified and efficient algorithms, at manageable effort

https://github.com/lammich/isabelle_llvm