Generating Verified LLVM from Isabelle/HOL

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- This talk: towards faster verified algorithms at manageable effort

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procedure AUGMENT(g, f, p)
  $c_p \leftarrow \min\{g_f(u,v) \mid (u,v) \in p\}$
  for all $(u,v) \in p$ **do**
    if $(u,v) \in g$ **then** $f(u,v) \leftarrow f(u,v) + c_p$
    else $f(v,u) \leftarrow f(v,u) - c_p$
  return f

procedure EDMONDS-KARP(g, s, t)
  $f \leftarrow \lambda(u,v).\ 0$
  while exists augmenting path in g_f **do**
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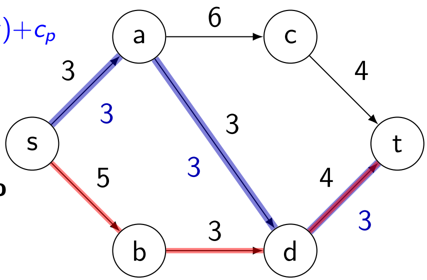

g: flow network s, t: source, target g_f: residual network

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Theorem (Ford-Fulkerson)

For a flow network g and flow f, the following 3 statements are equivalent

1. *f is a maximum flow*
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using basic concepts such as numbers, sets, and graphs. □

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- Implementations used for different parts must fit together!

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shortest-path-spec

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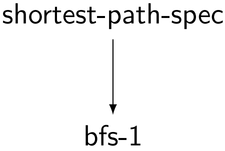

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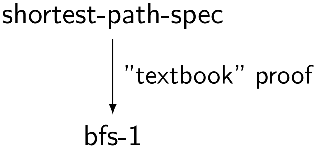

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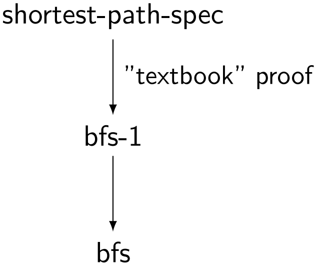

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↓ "textbook" proof

bfs-1

↓ graph $\rightarrow$ adj.-list
queue $\rightarrow$ ring-buffer

bfs

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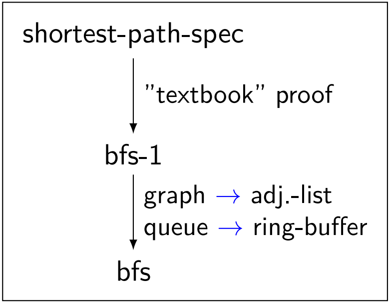

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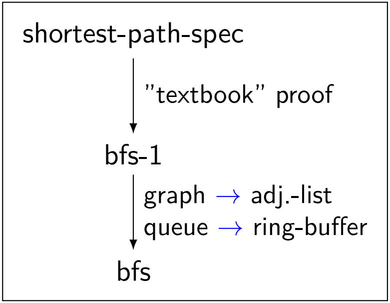

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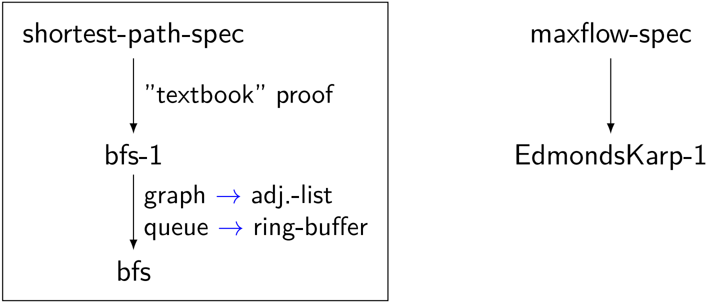

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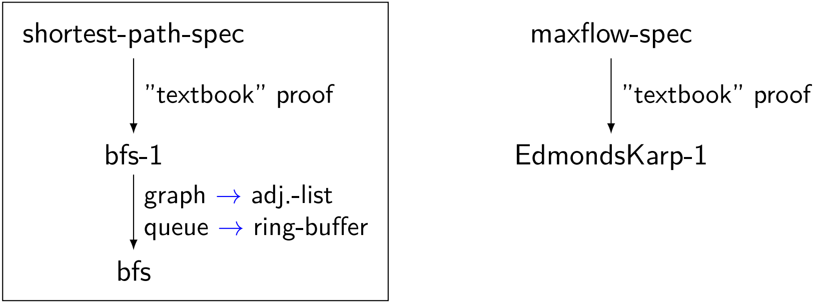

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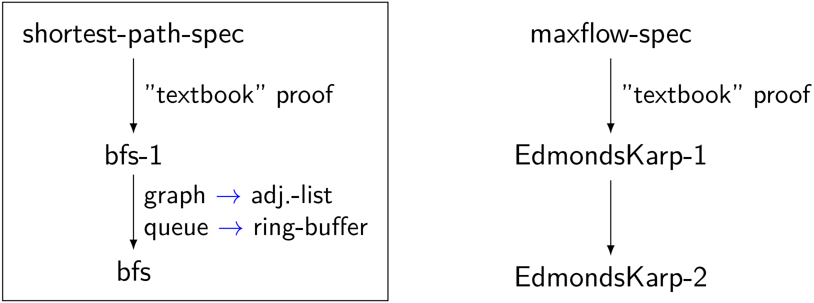

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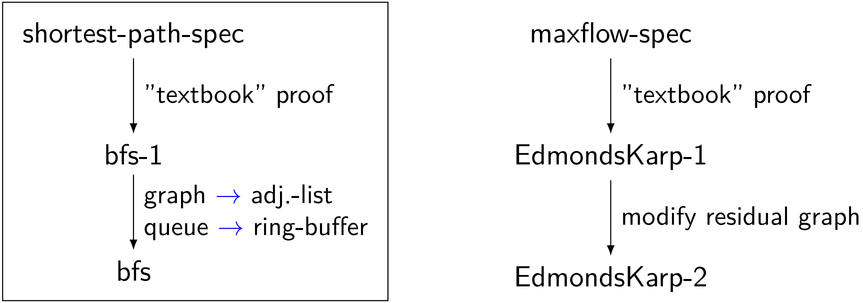

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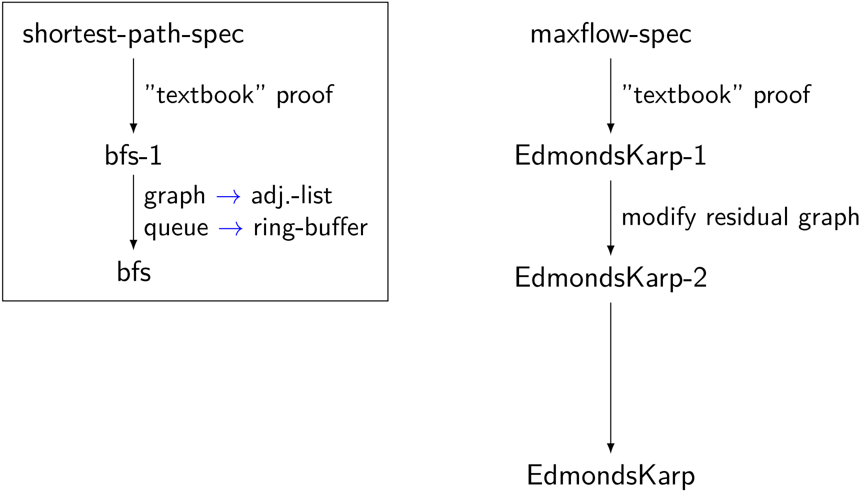

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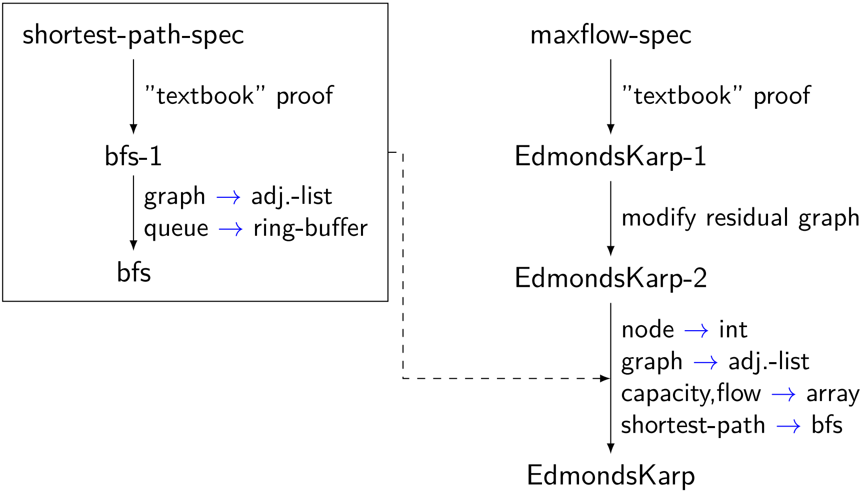

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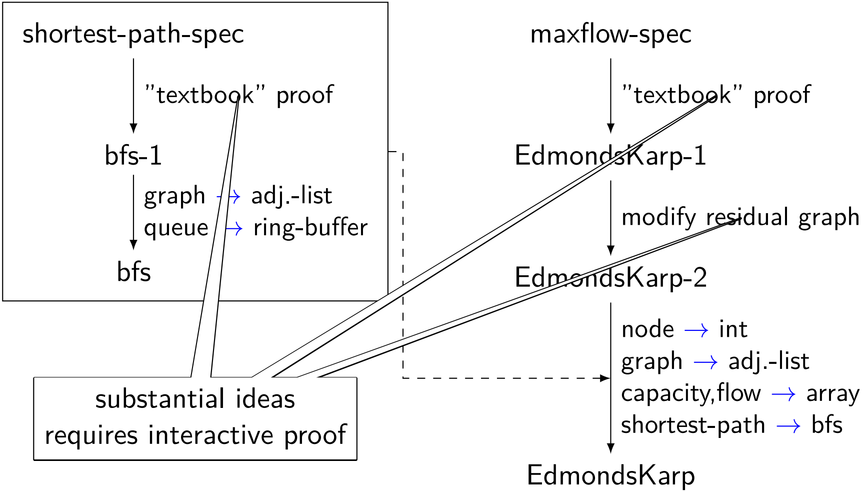

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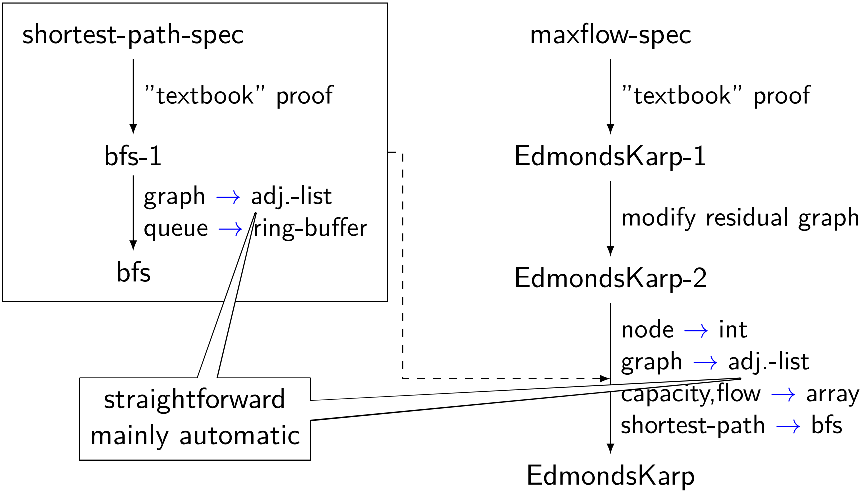

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  - Network flow (Push-Relabel and Edmonds Karp)

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- Automated transition from NRES to HEAP
  - automatic data refinement (e.g. integer by int64)
  - automatic placement on heap (e.g. list by array)
  - some in-bound proof obligations left to user

Code Generation

Translate HEAP to compilable code

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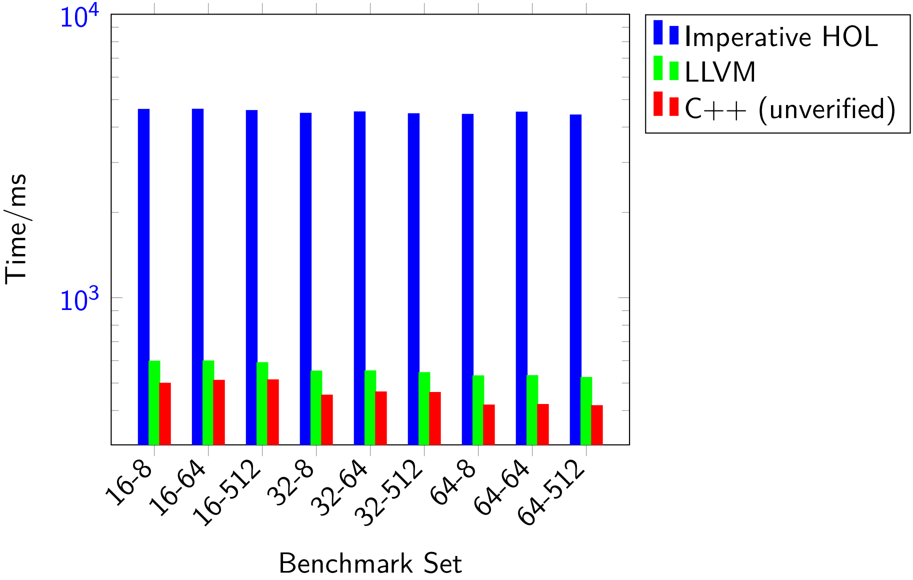
1. Imperative-HOL:
    - based on Isabelle's code generator
    - OCaml,SML,Haskell,Scala (using imp. features)
    - results cannot compete with optimized C/C++

Code Generation

Translate HEAP to compilable code

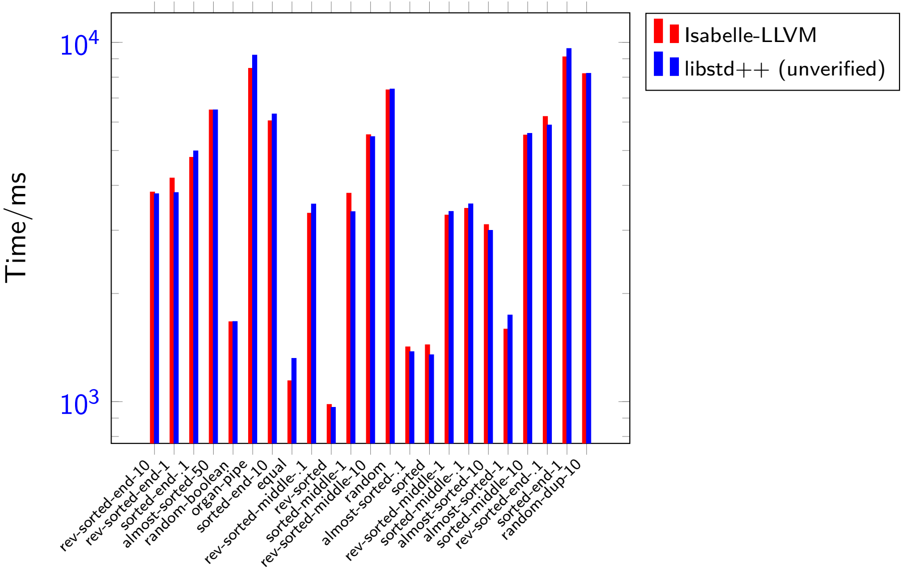
1. Imperative-HOL:
   - based on Isabelle's code generator
   - OCaml,SML,Haskell,Scala (using imp. features)
   - results cannot compete with optimized C/C++
2. NEW!: Isabelle-LLVM
   - shallow embedding of fragment of LLVM-IR
   - pretty-print to actual LLVM IR text
   - then use LLVM optimizer and compiler
   - faster programs
   - thinner (unverified) compilation layer

Knuth Morris Pratt



Execute *a-l* benchmark set from StringBench. Stop at first match.

Verified Introsort Algorithm



Sorting $100 \cdot 10^6$ `uint64`s on Intel Core i7-8665U CPU, 32GiB RAM.

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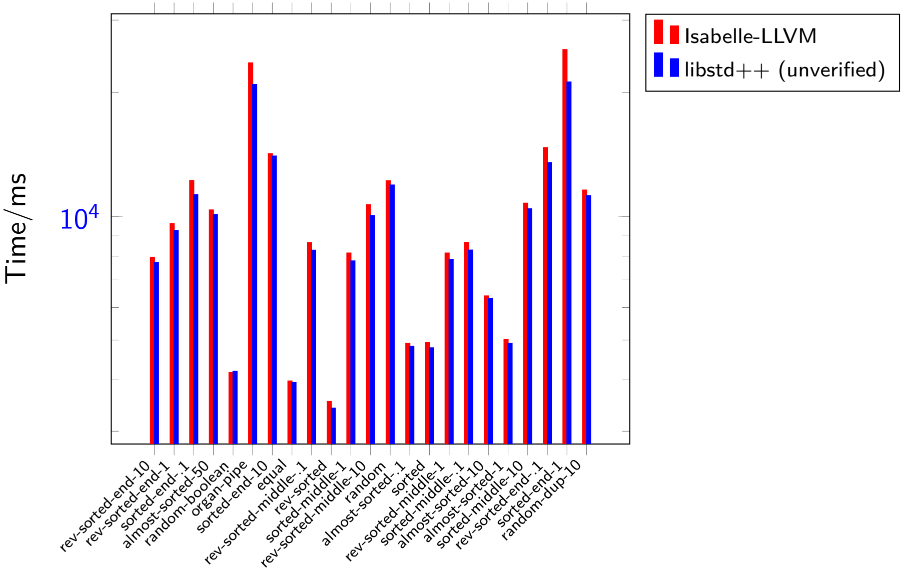
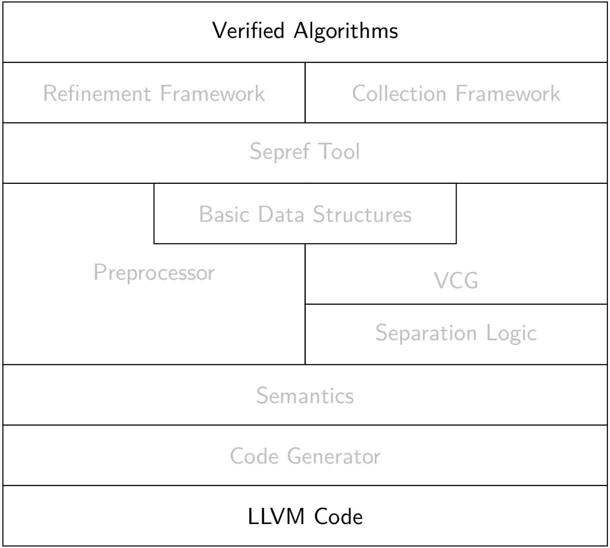


Sorting $100 \cdot 10^6$ `uint64s` on AMD Opteron 6176 24 core, 128GiB RAM.

Isabelle-LLVM: Overview



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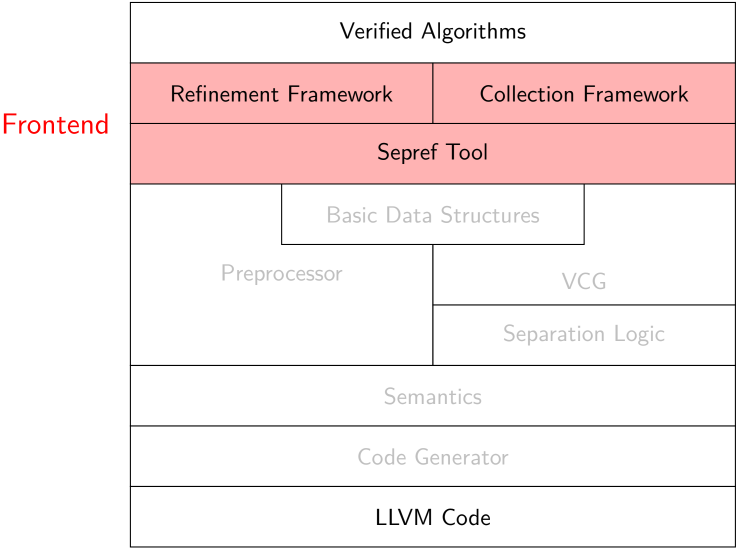

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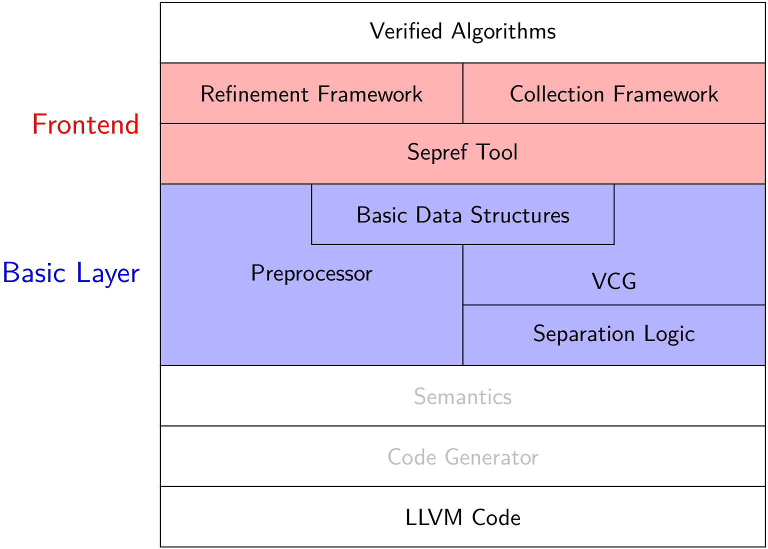

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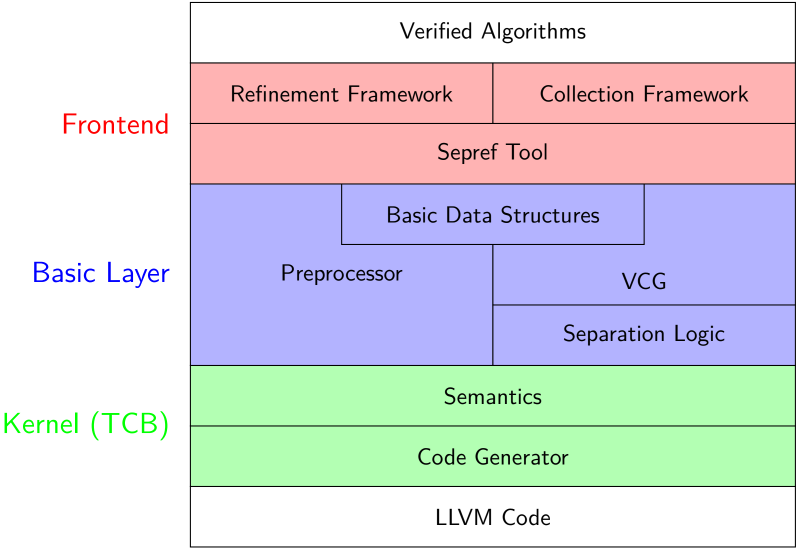

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- We don't need to formalize all of LLVM!
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- Trade-off
  - complexity of semantics vs. trusted steps in code generator
- Our choice:
  - rather simple semantics
  - code generator does some translations

Basics

- LLVM operations described in state/error monad

  α llM = llM (run: memory $\Rightarrow$ α mres)
  α mres = NTERM | FAIL | SUCC α memory

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- Recursion via fixed-point

```
llc_while b f s0 = fixp (λW s.
    do {
      ctd ← b s;
      if ctd≠0 then do {s ← f s; W s} else return s
    }
  ) s0
```

Shallow Embedding

```
fib:: 64 word ⇒ 64 word llM
fib n = do {
  t ← ll_icmp_ule n 1;
  llc_if t
    (return n)
    (do {
      n₁ ← ll_sub n 1;
      a  ← fib n₁;
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Code Generation

compiling control flow + pretty printing

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```

```
define i64 @fib(i64 %x) {
  start:
    %t = icmp ule i64 %x, 1
    br i1 %t, label %then, label %else
  then:
    br label %ctd_if
  else:
    %n_1 = sub i64 %x, 1
    %a   = call i64 @fib (i64 %n_1)
    %n_2 = sub i64 %x, 2
    %b   = call i64 @fib (i64 %n_2)
    %c   = add i64 %a, %b
    br label %ctd_if
  ctd_if:
    %x1a = phi i64 [%x,%then], [%c,%else]
    ret i64 %x1a }
```

Memory Model

- Inspired by CompCert v1. But with structured values.

  memory = block list   block = val list option
  val = n word | ptr | val×val
  rptr = NULL | ADDR nat nat (dir list)   dir = FST | SND

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- Typeclass llvm_rep: shallow to deep embedding

  to_val :: 'a ⇒ val
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- Shallow pointers carry phantom type

  'a ptr = PTR rptr

Example: malloc

```
allocn (v::val) (s::nat) = do {
  bs ← get;
  set (bs@[Some (replicate s v)]);
  return (ADDR |bs| 0 []) }
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ll_malloc (s::n word) :: 'a ptr = do {
  assert (unat n > 0); – Disallow empty malloc
  r ← allocn (to_val (init::'a)) (unat n);
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- Code generator maps ll_malloc to libc's calloc.
  - out-of-memory: terminate in defined way exit(1)

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  - Define recursive functions for fixed points

Example: Preprocessing Euclid's Algorithm

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euclid :: 64 word ⇒ 64 word ⇒ 64 word
euclid a b = do {
  (a,b) ← llc_while
    (λ(a,b) ⇒ ll_cmp (a ≠ b))
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preprocessor defines function euclid_0 and proves

```
euclid a b = do {
   ab ← ll_insert₁ init a; ab ← ll_insert₂ ab b;
   ab ← euclid₀ ab;
   ll_extract₁ ab }
euclid₀ s = do {
 a ← ll_extract₁ s;
 b ← ll_extract₂ s;
 ctd ← ll_icmp_ne a b;
 llc_if ctd do {...; euclid₀ ...} }
```

Reasoning about LLVM Programs

- Separation Logic
  - Hoare-triples

    α :: memory $\rightarrow$ amemory :: sep_algebra
    wp c Q s $=$ $\exists$r s$'$. run c s $=$ SUCC r s$'$ $\wedge$ Q r (α s$'$)
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  - memory primitives
    p$\mapsto$x – p points to value x
    m_tag n p – ownership of block (not its contents)

    range $\{i_1, \ldots, i\_n\}$ f p $=$ (p$+i_1$)$\mapsto$(f i_1) $*$ ... $*$ (p$+$i_n)$\mapsto$(f i_n)

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  - rules for commands

    b $\neq$ 0 $\Longrightarrow$ $\models$ {$\square$} ll_udiv a b {λr. r = a div b}
    $\models$ {p$\mapsto$x} ll_load p {λr. r=x $*$ p$\mapsto$x}
    $\models$ {n$\neq$0} ll_malloc n {λp. range {0..<n} (λ_. init) p $*$ m_tag n p}
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- Automation: VCG, frame inference, heuristics to discharge VCs
- Basic Data Structures: signed/unsigned integers, Booleans, arrays

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lemma

$\models$ {uint$_{64}$ a a$_{\dagger}$ $*$ uint$_{64}$ b b$_{\dagger}$ $*$ 0<a $*$ 0<b} euclid a$_{\dagger}$ b$_{\dagger}$ {λr$_{\dagger}$. uint$_{64}$ (gcd a b) r$_{\dagger}$}

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by (simp_all add: gcd_diff1 gcd_diff1′)

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    - need to be amended if they use arbitrary-precision integers

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    - we adapted it for LLVM
  - existing proofs can be re-used
    - need to be amended if they use arbitrary-precision integers
- Collections Framework
  - provides data structures
  - we ported some to LLVM (work in progress)
    - dense sets/maps of integers (by array)
    - heaps, indexed heaps
    - two-watched-literals for BCP
    - graphs (by adjacency lists)
    - ...

Example: Binary Search

```
definition bin_search xs x = do {
  (l,h) ← WHILEIT (bin_search_invar xs x)
    (λ(l,h). l<h)
    (λ(l,h). do {
      ASSERT (l<length xs ∧ h≤length xs ∧ l≤h);
      let m = l + (h−l) div 2;
      if xs!m < x then RETURN (m+1,h) else RETURN (l,m)
    })
    (0,length xs);
  RETURN l
}
```

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    (0,length xs);
  RETURN l
}

lemma bin_search_correct:
  sorted xs ⟹ bin_search xs x ≤ SPEC (λi. i=find_index (λy. x≤y) xs)
```

Example: Binary Search — Refinement

```
sepref_def bin_search_impl is uncurry bin_search
  :: (larray_assn' TYPE(size_t) (sint_assn' TYPE(elem_t)))^k
      * (sint_assn' TYPE(elem_t))^k
     → snat_assn' TYPE(size_t)
  unfolding bin_search_def
  apply (rule hfref_with_rdomI, annot_snat_const TYPE(size_t))
  by sepref
```

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```

sint_assn' sz — (mathematical) integers by *sz* bit integers
snat_assn' sz — natural numbers by *sz* bit integers
larray_assn' sz e — lists by arrays + *sz*-bit length, elements refined by *e*

Example: Binary Search — Refinement

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  unfolding bin_search_def
  apply (rule hfref_with_rdomI, annot_snat_const TYPE(size_t))
  by sepref

export_llvm bin_search_impl is int64_t bin_search(larray_t, elem_t)
defines
  typedef uint64_t elem_t;
  typedef struct { int64_t len; elem_t *data; } larray_t;
file code/bin_search.ll
```

Example: Binary Search — Generated Code

Produces LLVM code and header file:

```
typedef uint64_t elem_t;
typedef struct {
  int64_t len;
  elem_t*data;
} larray_t;

int64_t bin_search(larray_t,elem_t);
```

Conclusions

- Fast and verified algorithms
  - LLVM code generator
  - using Refinement Framework
  - manageable proof overhead
- Case studies
  - generate really fast, verified code
  - re-use existing proofs
- Current/future work
  - more complex algorithms
    - promising (preliminary) results for SAT-solver, Prim's algorithm
  - deeply embedded semantics
  - unify NRES and HEAP monads
  - generic Sepref (Imp-HOL, LLVM) $\times$ (nres, nres+time)

https://github.com/lammich/isabelle_llvm