Teaching Algorithms and Data Structures with a Proof Assistant (Invited Talk)

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Abstract
We report on a new course Verified Functional Data Structures and Algorithms taught at the Technical University of Munich. The course first introduces students to interactive theorem proving with the Isabelle proof assistant. Then it covers a range of standard data structures, in particular search trees and priority queues: it is shown how to express these data structures functionally and how to reason about their correctness and running time in Isabelle.

CCS Concepts: • Software and its engineering → Software verification; Formal software verification; Functional languages; • Applied computing → Education.

Keywords: Data structures, verification, teaching, Isabelle

ACM Reference Format:

1 Introduction
Teaching a course on programming language semantics and formalizing some of its meta-theory in the proof assistant Isabelle in the 1990s led me to fantasize about a Mechanized Semantics Textbook in Coq [2] and Concrete Semantics [13] in Isabelle [14].

This article is about a course (and a related book [11]) on verified functional data structures and algorithms taught annually since 2017 at TUM. Like Concrete Semantics, it is entirely based on Isabelle, but now the subject matter is algorithms instead of semantics. Of course correctness of algorithms, but also their running time complexity (including amortized analysis). The running time of a function \( f \) is expressed as a separate function \( T_f \) whose definition is directly derived from the definition of \( f \) but which counts computation steps. Alternatively one can also compute \( f \) and \( T_f \) simultaneously in a monadic framework [6, 10].

The next two sections present the actual contents of the course and the related book. The final section discusses generic aims and principles underlying the course.

2 Verified Textbook Algorithms
A recent survey [12] has underlined that the time is ripe for a textbook on verified algorithms: the authors surveyed which algorithms in the famous textbook by Cormen et al. [4] had been verified in an (automatic or interactive) theorem prover in the literature. It turns out that the only major omissions in the core of the book (excluding Selected Topics) are hashing including probabilities, Fibonacci heaps and van Emde Boas trees.

However, many of the algorithms in [4] are of an imperative nature. A second source of inspiration for us was Okasaki’s influential book Purely Functional Data Structures [15].

3 The Course
3.1 Format
The course is open to both bachelor’s and master’s students. There is one 1.5 hours lecture and one 1.5 hours lab session every week for 14 weeks. There is one homework sheet per week. The course is worth 5 ECTS points, with 30 ECTS which lead to the two mechanized semantics textbooks Software Foundations in Coq [2] and Concrete Semantics in Isabelle [14].

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1 Meanwhile a series of four volumes, the second of which is dedicated to programming language semantics [17], the third to functional algorithms [1]
points being the average workload per semester. The students must have some background in discrete maths (including an introduction to proofs), algorithms and functional programming.\(^2\)

The whole course utilizes the proof assistant Isabelle: in the lectures, the lab sessions and the homework. The final grade is a roughly equal combination of homework and a final exam. The 2020 exam was online, also using Isabelle, previous exams were paper-based.

### 3.2 Contents

The first 5 weeks of the course are dedicated to an introduction to Isabelle. The remaining 9 lectures are dedicated to data structures and algorithms. They are typically concerned with sorting, search trees, priority queues and amortized analysis. The topics are selected from chapters of a book in the making [11] with the following contents:

- Sorting: insertion sort, quicksort, top-down and bottom-up mergesorts
- Selection in linear time
- Binary trees
- Search trees:
  - 2-3 trees, red-black trees, AVL trees , Braun trees, tries, join-based \(\cup, \cap\) and \(\setminus\)
- Huffman’s algorithm
- Priority queues:
  - leftist heaps, Braun trees, binomial heaps
- Dynamic programming
- Amortized analysis:
  - splay trees, skew heaps, pairing heaps

Although suitable for teaching, the book is also meant as the definitive reference for precise correctness and complexity proofs: for some of the functional data structures in the book, e.g. Braun trees, precise proofs were missing in the literature (before the authors’ own recent work that is the basis of the book).

This is a live book that is meant to grow. New chapters are already in preparation, e.g. about the Hood-Melville queue, and the list of authors will grow. You could become a co-author yourself!

### 4 Aims and Principles

There are the obvious concrete aims: teach the algorithms themselves, their functional formulation and their correctness and running time proofs. Equally important are certain abstract aims that we discuss below.

#### Algorithms are Logic

Algorithms are often taught with little emphasis on formalities and correctness proofs. An important aim of this course is to instill into the student’s mind the understanding that one can reason about the correctness and complexity of algorithms expressed in a functional style just as easily as about numbers. Not just in principle, but with the help of a proof assistant also in practice.

The remaining aims and principles are independent of the subject matter and apply to any course utilizing an interactive theorem prover.

**No More LSD Trip Proofs!**

Computer science students frequently lack the training to write coherent proofs. Next to the algorithms, the central aim of the course is to teach the students the art of precise logical proofs. I believe that this goal is best reached if the following two principles are followed.

**Teach Proofs, Not Proof Scripts**

Most theorem provers provide a scripting language for writing proofs, typically as sequences of commands. Such proofs are for machines, not for humans. They do not show what property is being proved at each point. They are like assembly language programs. They do not convey ideas. I believe that proper proofs are best taught in some high-level structured proof language as offered, for example, by Mizar [5] and Isabelle [18].

Isabelle’s structured proof language Isar is used for most of the course. It is close to the informal language of mathematics and allows a smooth transition in the presentation style during the course: from Isar proofs on the machine to more traditional proofs on the blackboard (see below).

**Teach Proofs, Not Logic**

Mathematicians have been doing this successfully for a long time. To be provocative: proof systems like natural deduction belong in logic courses, where the fine structure of logic is studied. But constructing large proofs by single step inferences in some proof system is a straightjacket. Application-oriented courses (remember: the course topic is not logic!) should reason modulo logic: if the student believes that \(A\) together with \(B\) implies \(C\), she should be able to just write

\[
\text{from } A \text{ and } B \text{ conclude } C \text{ by hammer}
\]

where hammer is some suitable proof method of the underlying proof assistant. Isar allows exactly that, and Isabelle offers a number of automatic hammers for this purpose, in particular the connection to powerful external automatic provers [3]. The motto is: *Do not let logic dominate your thinking, let automatic provers take care of logic.*

It has to be acknowledged that using a theorem prover does not guarantee the intended learning outcome. Proof automation prevents frustration but also permits students to create proofs without understanding the underlying logical principles, e.g. induction.
Teach the Subject Matter, Not the Proof Assistant

I believe that teaching a course with a proof assistant is most convincing for the students if it focusses on the subject matter (here: algorithms and data structures) rather than the proof assistant. It is inevitable that in the beginning the proof assistant needs to be introduced (in our case 5 out of 14 weeks) and that for most of the course the students will struggle with it. But the more it can be deemphasized in the later lectures on the subject matter, the better. This leads to another principle:

Do Not Let the Proof Assistant Dominate Your Presentation

In the beginning of the course, when introducing the proof assistant, it is essential to demonstrate the interaction with the proof assistant in class for long periods.

During the second part of the course I gradually move to conventional presentations based on slides and the blackboard, although I never completely abandon Isabelle. I believe that slides (with animations) and the blackboard are better suited to explain many concepts and proofs than an Isabelle demo is. I rely on Isabelle’s \TeX \textit{generation facility} that supports prettyprinting of definitions and theorems from Isabelle theories on slides without having to type them in a second time (and getting them wrong).

When moving to the blackboard for developing proofs, I initially stick closely to Isar to phrase these proofs. As the students become more comfortable with Isar, I begin taking more and more liberties on the blackboard, moving towards informal proofs. The aim is to strengthen the students’ ability to bridge the gap between formal and informal proofs. While the lecture presentations gradually move away from Isabelle, the homework still is Isabelle-based and the students have to convert their intuition into Isabelle proofs.

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References